BAKER'S

RAILWAY ENGINEERING

AND

EARTHWORK

J. Matheson
Amsterdam Canal Work
Holland-op-Zijn Smalst
Beverwijk
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1865

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RAILWAY ENGINEERING;

OR,

FIELD WORK

PREPARATORY TO THE CONSTRUCTION OF RAILWAYS:

CONTAINING

THE ORIGINAL AND MOST APPROVED METHODS OF LAYING OUT RAILWAY CURVES, AND OF SETTING OUT THE WIDTHS OF THE CUTTINGS AND EMBANKMENTS, ETC.;

A

General Table

FOR THE

CALCULATION OF EARTHWORKS

OF

RAILWAY CANALS, ETC.

WITH TWO AUXILIARY TABLES;

ALSO,

TUNNELLING, AND INVESTIGATION OF THE FORMULA FOR THE SUPER-ELEVATION OF THE EXTERIOR RAIL IN CURVES.

BY

T. BAKER, C.E.

AUTHOR OF

" A System of Surveying by the Theodolite," and " Railway Engineering," in the Ninth Edition of Nesbit's Surveying ; " Integration of Differentials ;" &c. &c.

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Recat B

TO

PETER BARLOW, ESQ.

F.R.S., M. INST. C. E., ETC. ETC.

LATE PROFESSOR OF MATHEMATICS IN THE ROYAL MILITARY ACADEMY, WOOLWICH,

AND ONE OF THE COMMISSIONERS OF THE RAILWAY GAUGES,

AS

A MARK OF ESTEEM

FOR HIS DISINTERESTED KINDNESS,

AND

IN ADMIRATION OF HIS TALENTS

AS AN ORIGINAL MATHEMATICIAN AND PHILOSOPHER,

This Work is Dedicated,

WITH THE HONOUR OF HIS PERMISSION,

BY HIS MOST DEVOTED SERVANT,

THE AUTHOR.

PREFACE.

THE matter contained in the following pages, with the exception of a few additions and improvements, was drawn up about twenty-five years ago for private use; its publication, at the present time, having been delayed at least two years, for reasons, which it will be unnecessary here to state.

The chief part of SECTION I., On the Different Methods of laying out Railway Curves on the ground, was communicated to my friend the late professor Leybourn, of the R. M. College, Sandhurst, in 1824, and afterwards published by him in the "Gentleman's Diary;" these Methods being adapted to all cases that can occur in practice, as well as to the different degrees of scientific skill possessed by surveyors and engineers. The Constructions, Formulæ, and Methods of laying out Compound, Serpentine, and Deviation Curves, with Tables of Offsets for Railway Curves, were also placed in his hands; but his death prevented their publication.

As these Methods of laying out Railway Curves, especially two of them, have been generally adopted in practice, from the commencement of the Stockton and Darlington Railway to the present time, and as several other gentlemen, during the last two or three years, have published on the same subject, most of whom have given either all or part of my methods, I think it right to claim the Invention of them. See Remarks on the Invention of the Methods of laying out Railway Curves, p. 29.

The gentlemen that have published on this subject are, Law, Castle, Rankine, and Hill (see page 31). I have lately seen similar publications by Heald, Brodie, Gardner, May, and others. Almost all these authors give their formulæ and rules for laying out curves without investigation; some of them have given an unnecessary profusion of formulæ, which involve the subject they pretend to explain, in such a degree of obscurity as must be very perplexing to students, and repulsive to those engineers who have been accustomed to use my methods, *in which I anticipated, above twenty years ago, all that has since been done by these authors, at least as far as real practical utility is concerned.*

In SECTION II. I have given Methods of setting out the widths of ground for Railway Cuttings, on horizontal, laterally sloping, and uneven ground, with original Formulæ connected therewith; superseding, in most cases, the unscientific methods of approximation given for the same purpose used by many engineers. This Section concludes with methods of calculating the quantity of land required for projected and intended Railways.

In SECTION III. I have treated extensively on finding the Con-

tents of Railway Cuttings, both for preliminary Estimates from the depths, and for actual Cuttings from Sectional Areas, by means of a General and two Auxiliary Tables, on one folding sheet at the end of the work. — *The Method here given of finding the Contents of Cuttings from Sectional Areas is, as far as I know, the only one mathematically correct, yet published*, having been prepared for private use, many years ago.

The authors who have published Tables on this subject are Sir John M'Neill, Mr. Bidder, Mr. Bashforth, Messrs. Sibley and Rutherford, Mr. Huntingdon, Mr. Law, and others; some of whose Tables are voluminous, and most of them well adapted for finding the Contents of Cuttings for preliminary Estimates; but none of them are accompanied with directions for finding the Contents from Sectional Areas, which is the most important part of the use of such Tables, except Mr. Bashforth's: *but his method of applying them is erroneous*. See pages 41 and 54. This Section contains Investigations of the Methods of constructing the Tables, and of using them for finding the Contents of Cuttings; and concludes with investigating and pointing out the errors of Mr. Bashforth and others.

In SECTION IV. I have given Methods of setting out the Earthwork of Tunnels, with a Table of the Dimensions of several existing Tunnels.

In SECTION V. I have given the Investigation and Application of the celebrated Formula for the Superelevation of the Exterior Rail in Railway Curves, which is, I believe, due to *Pambour*.

The matter contained in this Work is drawn up in a manner which, I trust, will be easily understood by any surveyor or engineer; Examples, wrought out at length, being given in every case, the mathematical Investigations of the Problems, &c. being kept separate from the practical parts of the Work, the whole of which, except a few trifling alterations, form part of a System of Railway Engineering, which I added to the ninth edition of Nesbit's Surveying, lately published.

In conclusion, I can truly say that the whole of the matter contained in this small volume, with some few trifling exceptions, was originally drawn up by me; the most essential parts of which have either been published or communicated to my friends for periods varying from ten to twenty-five years: no author, on the subjects here treated of, having preceded me, whose results appeared to be of a character adapted to this work, excepting one of *Pambour's* as already noticed; and a small part of the investigation of the rules for finding the contents of earthworks which has been given, as is well known, by various authors.

T. BAKER.

Vauxhall Street, London, 1848.

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RAILWAY ENGINEERING;

OR

FIELD WORK

PREPARATORY TO THE CONSTRUCTION OF RAILWAYS.

SECTION I.

THE METHODS OF LAYING OUT RAILWAY CURVES ON THE GROUND.

On the Use of Curves in Railways in general.

THE use of curves in railways is absolutely necessary on account of the natural unevenness of the earth's surface, it being desirable to attain the nearest possible practical level by avoiding hills, crags, mountains, &c., by winding round them by means of curves. Curves are also equally necessary in avoiding other natural and artificial obstructions, in many cases not materially affecting the level of the line; as lakes, swamps, the windings of sea-coasts, and of rivers; also cities, towns, villages, parks, pleasure-grounds, &c. In this manner a great saving is effected in the expense of extensive excavations, embankments, tunnels, viaducts, &c., as well as the expense of the severance of valuable property, which would otherwise be required. Besides, it is frequently desirable to make a winding railway, in order that it may embrace in its route some important city, town, harbour, &c., or make a junction with another railway. Straight lines in railways are, however, much to be preferred to curves, and are therefore adopted as far as is practicably possible, and are first set

out as bases for further operations; every curve being, in all cases, excepting at the terminus of a line, subordinate to two consecutive straight portions of the line, which are tangents to it at its commencement and termination. The curve adopted in practice is always an arc of a circle; or two, three, or more consecutive circular arcs, having a common tangent at their point or points of junction, as in the case of the compound curve; or two or more circular arcs, having their convexities turned different ways, with a common tangent at their point or points of junction, as in the case of the serpentine or S curve.

NOTE. — The parabola, ellipse, cycloid, &c., might with propriety, as well as advantage, be adopted as the curves of railways in many cases; but their construction is attended with difficulty, and a near approach may be made to any of these curves by means of the compound curve.

(1.) *On the Different Methods of laying out Railway Curves.*

At first sight it would appear that one method of laying out railway curves would be amply sufficient; but it has been found in practice that both on account of the degree of scientific skill possessed by the surveyor or engineer, and of obstructions on the ground, as wood, rivers, buildings, &c., presenting themselves either on the concave or convex side of the curve, or swamps, bogs, quarries, &c., preventing the use of the surveying chain, that at least four methods of laying out the curve are requisite, which are given in the following Problems; also various methods of construction and formulæ connected with the laying out compound, serpentine, and deviation curves.

NOTE. — Other authors have given other methods of laying out curves, most of which are either impracticable or require complicated calculations. See Preface and Remarks on the invention of the methods of laying out railway curves, p. 29.

(2.) *On Mechanical Railway Curves, or Curve-rulers.*

Mechanical railway curves, sometimes called curve-rulers, or railway curves, are a series of segments of circles made of hard wood, as box or mahogany, or strong paste-board, with their radii in inches

marked on them. These curves commonly begin with a radius of $2\frac{1}{2}$ inches, and terminate with one of 160 inches or upwards; the smallest radii usually increase by $\frac{1}{2}$ inches up to 10 inches; then by inches up to 20 inches; afterwards by 2 inches up to 80, including all numbers ending in 5; lastly, by 5 inches, till near the end of the series, when the radii increase by 10 inches.

The annexed figure represents the railway curve-ruler of 16 inches radius. If the scale of plan, to which this curve-ruler is applied, be 5 chains to an inch, it will represent a curve or circular arc of $16 \times 5 = 80$ chains $= 1$ mile radius. If the scale of the plan be 12 chains to an inch, it will represent an arc of $16 \times 12 = 192$ chains $= 2$ miles and $3\frac{1}{5}$ furlongs radius, and similarly for other scales.

(3.) *The limit of the radii of railway curves.*

One mile, or 80 chains, is the least possible, or *minimum* limit of the radii of railway curves, as appointed by the British parliament; because, in curves of less radius, the centrifugal force, or tendency to motion in a tangental direction, of a railway train of great velocity, is so much increased as to endanger its running off the line, on the convex side of the curve. This limitation is dispensed with, when the curve is at or near one of the *termini* of the railway, or at or near a principal station, where the speed of the train must be gradually diminished, before it reaches the curve, for the purpose of halting. Curves of less than a mile radius may, however, be safely admitted by giving a proper superelevation to the exterior rail to counterbalance the centrifugal force. (See Formula, Section V.)

PROBLEM I.

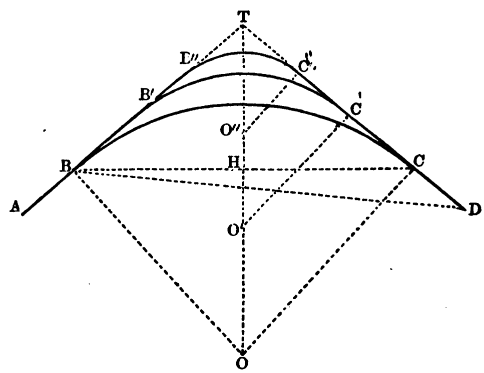
CASE I.

To determine the Radius of a Railway Curve mechanically, that is, by the Curve Rulers; the positions of the straight, or tangental portions of the Railway being given.

This problem admits of an indefinite number of solutions, but the curve adopted is generally that which falls on ground presenting the fewest natural or artificial obstructions, provided its radius be equal to, or exceed 80 chains. (See Art. 3.)

Let A B, C D, be two straight portions of a railway, the positions of which are given by an accurate plan; and which, when prolonged, meet at T. Apply several of the series of curve-rulers (see Art. 2.) to touch the line A B, without cutting it, at the points B, B′, B″, &c. and also to touch C D, in like manner, at the points C, C′, C″, &c.;

and let B C, B′ C′, B″ C″, &c., be the curves thus obtained. Then whichsoever of these curves may appear, on a careful consideration, to be least obstructed by hills, lakes, rivers, buildings, &c., must be adopted as part of the line of railway, provided its radius be equal to, or exceed the required limit. The radius is determined by multiplying the number on the curve-ruler, by the number of chains per inch on the scale of the plan.



Example. If the curve B C be drawn by the curve-ruler, numbered 22, and be adopted as part of the line, and the scale of the plan be 5 chains to an inch, required the radius of the curve.

$22 \times 5 = 110$ chains = 1 mile, 3 fur., the radius required.

NOTE. — By this method the radii of railway curves are commonly found in practice; but since a circular arc of large radius apparently coincides with its tangent for a considerable distance, it is difficult, in this manner, to determine the exact starting points of the curve; besides, if the plan be not strictly accurate, as is too often the case, the positions of the tangents cannot be said to be given, and therefore this method is out of the question, except in roughly determining the radius and starting points of the curve, so that it may fall, in its progress, on the most favourable ground.

CASE II.

To determine the Radius of a Railway Curve geometrically, the Starting Point B *being given.*

Prolong A B, D C, till they meet at T; bisect the angle A T D by the line T O; and at B draw B O perpendicular to A T, meeting T O in O. Then O is the centre, and B O the required radius of the curve B C.

OR,

Make T C = T B; and draw the perpendiculars B O, C O, meeting in O. Then B O, or C O, is the radius of the curve.

In the same manner, the other radii C′ O′, C″ O″, &c., of the curves B′ C′, B″ C″, &c., may be found.

NOTE. — The truth of the two preceding methods of finding the radius follows from the nature of the tangents.

CASE III.

To find the Radius of the Curve, by taking Dimensions on the Ground, the Map being known to be inaccurate, and the Starting Point B *being given.*

Range the tangents A B, D C, till they meet at T; measure B T; then measure on T D till T C = B T; also measure B C. Then $BO = \frac{1}{2}BC \cdot BT \div \sqrt{BT^2 - \frac{1}{4}BC^2}$ * = radius required.

Ex. Let B T = 82.50 chains, and B C = 132.00 chains. Then $BO = \frac{1}{2}(132 \times 82.5) \div \sqrt{(82.5)^2 - \frac{1}{4}(132)^2} = 5445 \div 49.5 = 110$ chains, the radius required.

OR, BY TRIGONOMETRY.

Range the tangents A B, D C, till they meet at T, as before, then measure B T, and take the angle B T C, $\frac{1}{2}$ of which is the ∠ B T O; then in the triangle B T O, right-angled at B, are given B T and the ∠ B T O to find B O; whence by trigonometry,

cot. ∠ B T O : rad. :: T C : B O the radius required.

Ex. Let B T = 82.50 chains, and the angle B T C = 106° 16′.

Then cot. $\frac{1}{2}$ ∠ B T C = cot. ∠ B T O, 53° 8′ -	9.87501
: rad. or sin. 90° - - - -	10.00000
:: B T = 82.50 chains - - -	1.91645
: B O = 110 chains, the required radius -	2.04144

CASE IV.

To find the Radius of the Curve, when the Plan is known to be inaccurate, and the Tangents A B, D C, *cannot be prolonged to meet at* T *on account of obstructions, the Starting Point* B *of the Curve being given.*

From B measure a line B D, on the most convenient ground, to meet C D at D, taking the angles T B D, T D B; the sum of which taken from 180° gives the ∠ B T D, whence by trigonometry,

sin. ∠ B T D : B D :: sin. ∠ T D B : B T.

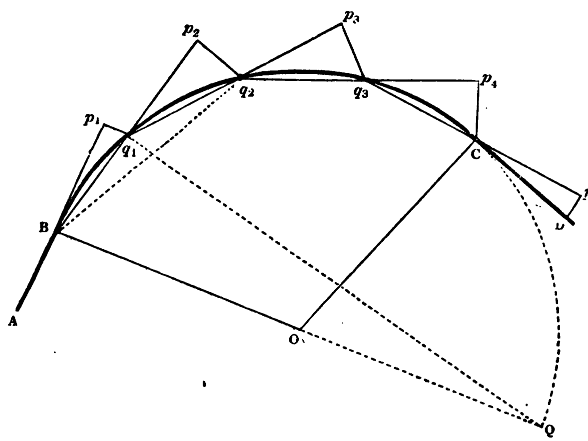
Now in the right-angled triangle B T O are given B T and the angle B T O = $\frac{1}{2}$∠ B T D, to find the radius B O, which may be done as in Case III.

* *Demonstration.* — In the △ B T H, $TH = \sqrt{BT^2 - BH^2} = \sqrt{BT^2 - \frac{1}{4}BC^2}$, and by the similar △s B H T, B H O, $\sqrt{BT^2 - \frac{1}{4}BC^2}$ (= T H) : B T :: $\frac{1}{2}$ B C (= B H) : B O = $\frac{1}{2}BC \cdot BT \div \sqrt{BT^2 - \frac{1}{4}BC^2}$. *Q. E. D.*

PROBLEM II.

To lay out a Railway Curve by the common Method.

Let A B, C D be two straight or tangental portions of a railway, the points B, C being required to be joined by a circular curve B C, to which A B, C D shall be tangents at the points B and C; and let B O be the radius of the curve, which is supposed to be deter-



mined by one or other of the cases in Problem I. accordingly as the map is found to be accurate or inaccurate. Put the radius B O $= r$; measure on the tangent A B prolonged the distance B $p_1 = 1$ chain, as is usual in practice; set off $p_1\, q_1 = \frac{1}{2\,r}$ * at right angles to B p_1; then q_1 is the first point in the curve. Through B q_1 measure the right line B $q_1\, p_2 =$ twice B $p_1 = 2$ chains, and set off $p_2\, q_2 = \frac{1}{r} =$ twice $p_1\, q_1$ at right angles to B $q_1\, p_2$; then q_2 is the second point in the curve, repeating the last operation till the curve reach the point C. Lastly, q_3 C p_5 being measured $= 2$ chains, the last offset p_5 D

* *Demonstration.* — Complete the semicircle B C Q, prolong B O to Q, and join q_1 Q, B q_2. Then because B p_1, B p_2, B q_2 are always, in practice, so very small compared with B Q that they nearly coincide with the curve, and consequently B p_1 is approximately $=$ B q_1 $=$ $q_1\, p_2$ $=$ $q_1\, q_2 = \frac{1}{2}$ B q_2; $\therefore$ by the nature of the circle the Δs B q_1 Q, B $p_1\, q_1$, B $p_2\, q_2$ are similar, and

$$\mathrm{B\,Q} : \mathrm{B}\,q_1 :: \mathrm{B}\,q_1 : p_1\,q_1 = \frac{\mathrm{B}\,q_1^2}{\mathrm{B\,Q}} = \frac{\mathrm{B}\,p_1^2}{\mathrm{B\,Q}} = \frac{\delta^2}{2\,r},$$

$$\text{also} \quad : \quad :: \mathrm{B}\,q_2 : p_2\,q_2 = \mathrm{B}\,q_1 \times \mathrm{B}\,q_2 = \frac{2\,\mathrm{B}\,p_1^2}{\mathrm{B\,Q}} = \frac{\delta^2}{r}.$$

B O $= \frac{1}{2}$ B Q being put $= r$, and B $p_1 =$ B $q_1 =$ &c. $= \delta$; and when $\delta = 1$ chain, as in practice, $p_1\, q_1 = \frac{1}{2\,r}$, and $p_2\, q_2 = p_3\, q_3 =$ &c. $= \frac{1}{r}$. *Q. E. D.*

will be found = the first offset $p_1 q_1$, or half the preceding one p_4 C, if the work be right. See the following notes and example.

NOTE 1. — Only a small part of the operation for a curve of great length is shown in the figure; but as the whole of the work, except the first and last offsets, is alike, to show more would be unnecessary.

NOTE 2. — The values of $\frac{1}{2r}$ for all radii, from 15 chains to 540, are given in Table No. 4. at the end of the work.

Ex. Let B O $= r =$ 80 chains = 1 mile; then $\frac{1}{2r} = \frac{1}{160}$ of a chain, which being multiplied by 792, the number of inches in 1 chain, gives $\frac{1 \times 792}{160} = 4.95$ inches, the first offset $p_1 q_1$; whence $4.95 \times 2 = 9.9$ inches, the second offset $p_2 q_2$.

Or, by Table No. 4. opposite 80, in the column marked Radius, &c. stands 4.95 inches, in the column marked Offsets, &c. which is the value of $p_1 q_1$, as above.

NOTE 3. — By this method the greater part of British, as well as foreign, railway curves, have been laid out, having been invented by the author about 25 years ago, when the Stockton and Darlington railway was laid out. It was eagerly adopted by railway surveyors, as it involves very little calculation, and does not require the use of an angular instrument. It is, however, defective in practice, on account of its requiring the coupling together of so very many short lines, as small errors will unavoidably creep in and multiply, and more especially so if the ground be rough, so that the curve has frequently to be retraced several times before it can be got right, especially if it be a long one. *This defect in the above method induced the author to prepare three other methods, which are given in the following Problems, and which he would recommend to engineers according to circumstances. See Observations on the invention of the methods of laying out railway curves at the end of these Problems.*

NOTE 4. — When the curve shall have been laid out with sufficient accuracy, the rods or quills that mark the ends of the offsets, q_1, q_2, &c. must be taken out, and their places supplied by strong wooden stumps, about 16 or 18 inches in length, and 1¼ inch square, each end of the curve being marked with three stumps, or one large one with a cross or some other conspicuous mark on it. The straight portions of the line must also be marked with stumps at the end of every chain.

PROBLEM III.

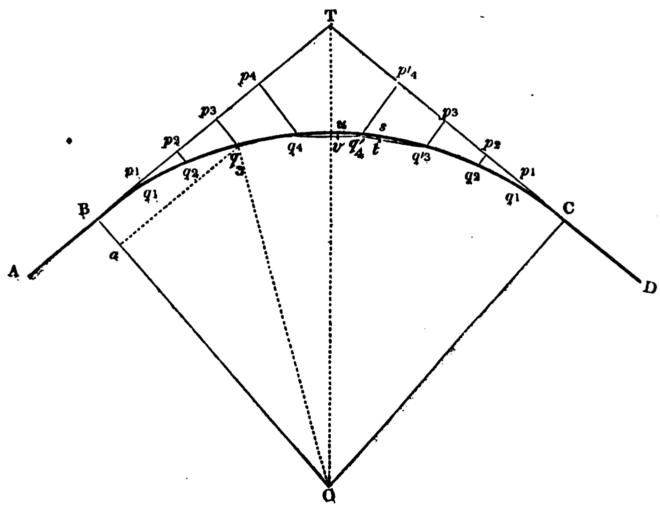
To lay out a Railway Curve by Ordinates or Offsets from its Tangents, no material Obstructions being supposed to exist on the convex Side of the Curve, to prevent the use of the Chain.

CASE I.

When the length of the Curve is less than ¼ of its Radius.

Let B C be the curve, A B, C D tangents thereto, which are ranged till they meet at T, and B O the radius of the curve, which

must be correctly determined by one or other of the cases in Prob. I. according to circumstances. Measure on B T the usual distance $B p_1 = 1$ chain, and lay off the offset $p_1 q_1 = \frac{1}{2r}$ (r being the radius of



the curve as before), then measure $p_1 p_2 = 1$ chain, and lay off $p_2 q_2 = \frac{2^2}{2r}$. The successive offsets, at the end of every chain being $\frac{1}{2r}, \frac{2^2}{2r}, \frac{3^2}{2r}, \frac{4^2}{2r}$, &c.*, or what amounts to the same thing, the

* *Demonstration.* — Draw the radius q_3 O, and $q_3 a \perp$ to O B. Put O B $= O q^3 = r$, and $q_3 a = p_3 B = \delta$; then $O a = \sqrt{r^2 - \delta^2}$, and $B a = p_3 q_3 = O B - O a = r - \sqrt{r^2 \delta^2}$. Now, if δ be taken successively $= 1^2, 2^2, 3^2$, &c. chains, the values of $p_1 q_1, p_2 q_2, p_3 q_3$, &c. will be respectively $r - \sqrt{r^2 - 1^2}$, $r - \sqrt{r^2 - 2^2}$, $r - \sqrt{r^2 - 3^2}$, &c. But as $1^2, 2^2, 3^2$, &c. are very small compared with r^2, within the limitation assigned to Case I. of this Problem, the values of $\sqrt{r^2 - 1^2}$, $\sqrt{r^2 - 2^2}$, &c. may be taken respectively $r - \frac{1^2}{2r}$, $r - \frac{2^2}{2r}$, &c. without material error, whence $p_1 q_1 = r - \left(r - \frac{1^2}{2r}\right) = \frac{1^2}{2r}$, $p^2 q^2 = r - \left(r - \frac{2^2}{2r}\right) = \frac{2^2}{2r}$, &c. But when the length of the curve exceeds $\frac{1}{4}$ of its radius, only 5 or 6 of the offsets ought to be taken in this manner, and the remainder, say, commencing with $p_7 q_7$, ought to be taken $= r - \sqrt{r^2 - 7^2}$, &c. Although in a curve of 80 chains' radius, the 10th offset by the contracted method exceeds the same offset by the correct method by only $\frac{1}{4}$ of a link out of $62\frac{1}{2}$ links. This difference, however, becomes gradually greater as the distance on the tangent approaches the 20th chain.

2d, 3rd, 4th, &c. offsets, are respectively 2^2, 3^2, 4^2, &c. times the first offset, or 4, 9, 16, &c. times the first one.

Having laid out the offsets in this manner, till the last one, which suppose to be $p_4\ q_4$ is either within, or very little more than, a chain from T, make $p'_4\ \mathrm{T} = p_4\ \mathrm{T}$, and lay out the same offsets in an inverted order on T C, as were laid out on B T, that is, beginning with the greatest first and ending with the least.

Ex. Let the radius of the curve be 80 chains, then $p_1\ q_1 = \frac{1 \times 792}{2 \times 80}$ $= 4.95$ inches; whence $p_2\ q_2 = 4 \times 4.95 = 19.8$ inches, $p_3\ q_3 = 9 \times 4.95 = 44.55$ inches $= 3$ ft. 8.55 in. &c., or the value of the first offset may be taken from the table at the end of the book.

To make the Distances of the Stumps equal.

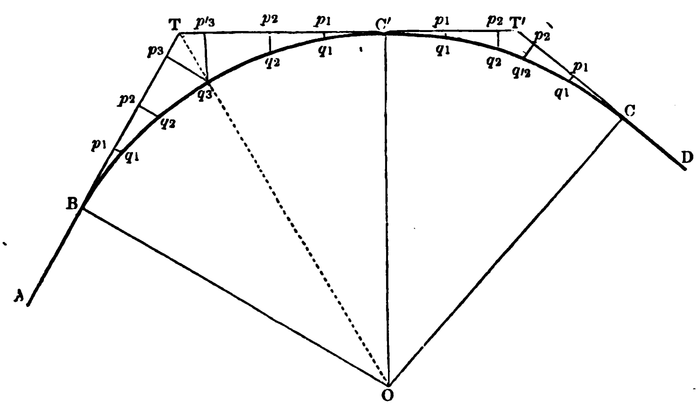
As it can rarely happen in practice that the distance $q_4\ q'_4$ will be $= 1$ chain, when this is the case, it will be better to set out the curve from C as was done from B: then, if the distance $q_4\ q'_4$ be less than 1 chain, suppose m links less, lay the chain from q'_4 to q'_3, and at the distance $q'_4\ t = m$, lay off the offset $t\ s = \frac{m\ (1-m)}{2\ r}$, then s will be the point for the stump, instead of q'_4, thus making $q'_4\ s = 1$ chain. This operation must be repeated between q_3 and q_2, q_2 and q_1, &c., the last point to be changed necessarily falling on the tangent C D. If the distance $q_4\ q'_4$ be greater than 1 chain by m links, lay the chain from q'_4 in the direction of q_4, and, at the distance of $q'_4\ v = m$ links, lay off the offset $v\ u = \frac{(1+m)\ m}{2\ r}$, then u will be the place for the stump; next lay the chain from q_3 in the direction of u, and repeat the previous operation: this must be done to the end of the curve, the last mark to be changed falling, in this case, within the curve. The stumps may now be put down as pointed out in Note 4. Prob. II.

Ex. 1. When the distance $q_4\ q'_4$ is less than 1 chain by 40 links, the radius of the curve being 80 chains. Here $t\ s = \frac{(1-m)\ m}{2\ r} = \frac{(1-.40) \times .40}{160}$ ch. $= \frac{.60 \times .40 \times 792}{160} = 1.188$ in. $= 1\frac{1}{5}$ inch nearly.

Ex. 2. When the distance $q_4\ q'_4$ is more than 1 chain by 40 links, and rad. $= 80$ chains. Here $v\ u = \frac{(1+m)\ m}{2\ r} = \frac{(1+.40) \times .40}{160}$ h. $= \frac{(1.40 \times .40) \times 792}{160} = 2.742$ in. $= 2\frac{3}{4}$ inches nearly.

CASE II. *When the Curve is any required Length.*

As it is inconvenient in practice to have the offsets greatly to exceed two chains in length ; if, therefore, the curve be a long one, the offsets may be confined within proper limits by dividing the curve into two or more parts, and introducing one or more additional tangents. In the annexed figure the curve B C is divided into two



parts in C′, at which point the tangent T C′ T′ is drawn, meeting the tangents B T, T′ C in T and T′, the tangents B T, T C′ being each taken $= B p_3 + \frac{1}{8} p_3 q_3$*, wherein $B p_3$ is taken the nearest whole number of chains to $\frac{1}{8}$ B O, and $p_3 q_3$ is the offset corresponding to $B p_3$. By thus taking the length of B T, T C′, the last offset on B T and the first one on T C will meet at q_3, the middle point of B C′, and therefore the distances of the ends of the offsets, that form the curve, will all be sufficiently near to equality, *i. e.* to one chain. Also by trigonometry, $BT : BO :: \text{rad.} : \tan. \angle BTO = \frac{1}{2} \angle BTC'$; whence the $\angle BTC'$ becomes known, and consequently the direction of the new tangent, T C′ T′ is also known. Having, therefore, determined B T and $\angle$ B T C′, the offsets may be laid out to $p_3 q_3$; and having made $T p_3 = T p'_3$, the same offsets may be laid out, in an inverted order on T C′; the order of the offsets being a second

* *Demonstration.*—By the similar triangles O B T, $q_3\ p_3$ T, $OB : Bp_3 + p_3T$ $(BT) :: p_3 q_3 : p_3 T$; but since $B p_8 = \frac{1}{n}$ O B, or the nearest whole number to it, and since p_3T must always be very small compared with Bp_3, it may be rejected; $\therefore OB : \frac{1}{n} OB$ (= very nearly to B T) $:: p_3 q_3 : p_3 T = \frac{1}{n} p_3 q_3$, wherein n may be taken any whole number, 8 being the most convenient, as in the example.

time inverted on C′ T′, and a third time on T′ C. If the distance of the middle offsets $q_2\, q'_2$ be less or greater than 1 chain, that distance, and consequently the following ones, must be made equal to 1 chain, as in the preceding case.

Ex. Let B O = 240 ch., then $B\,p_3 = \frac{1}{8}$ B O = 30 ch., $p_3\,q_3 = \frac{B\,p^2_3}{2\,BO} = \frac{30^2}{480} = 1.875$ ch.: whence B T = T C′ $= B\,p_3 + \frac{1}{8}\,p_3\,q_3 =$ $30 + \frac{1}{8}\,1.875 = 30.2344$ ch. $= 30.23\frac{1}{2}$ ch. nearly: and by trig.

As 30.2344 (B T) log.	- -	1.48050
: 240 (B O)	- - -	2.38021
:: rad.	- - - -	10.00000
: tan. $\frac{1}{2}\,\angle$ B T C′ =	82° 49′	10.89971
	2	
$\angle$ B T C =	165° 38′,	

whence the position of the tangent T C′ T′ becomes known; it will be unnecessary in this case to find $\angle$ T T′ C.

By table 4 the first offset on B T, viz. $p_1\,q_1 = 1{\cdot}65$ in., from which the other offsets may be found by multiplying by 4, 9, 16, &c. which may be laid out as in the last case, $p_3\,q_3$ being the 30th offset from B, and meeting the corresponding offset $p'_3\,q_3$ at q_3, the middle of the curve B C′. The method of making the distances equal on the seeond part C′ C of curve, if required, is the same as in Case I.

NOTE. — If the curve be a long one, it may be divided, in this manner, into 3, 4, or 5 parts, according to its length.

CASE III. *When the Length of the Curve exceeds the Limit adopted in* Case I., *but is considered too short to be divided into Two Parts, as in* Case II.

Let the curve be one of 80 chains radius, and between 32 and 38 chains in length, then to avoid the trouble of adding another tangent, the offsets beyond the 8th must be calculated from the following formulæ: —

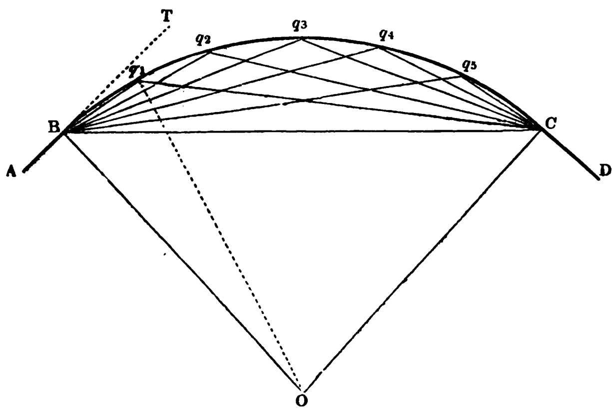
$$p_7\,q_7 = r - \sqrt{r^2 - 9^2}$$
$$p_8\,q_8 = r - \sqrt{r^2 - 10^2}$$
$$\&c. = \quad \&c.$$

If the radius of the curve be 160 chains, and its length about 50 or 60 chains, the offsets must be calculated, as above, beyond the 15th. The formula $\frac{1^2}{2\,r}, \frac{2^2}{2\,r}, \frac{3^2}{2\,r}$, &c. being a very near approximation to the true lengths of the offsets, within the limits assigned, but beyond these limits the errors of the offsets begin gradually to augment, till they become too considerable to be overlooked. See Demonstration to Case I.

PROBLEM IV.

To lay out the Curve, where Water, Swamps, Quarries, or other Obstructions prevent the use of the Chain, by Two Theodolites.

The position of the tangents A B, D C to the curve at its extremities B and C, its radius O B or O C, and the $\angle$B O C = supple-



ment of the angle made by the tangents when prolonged, being determined; join B C, and take several equidistant points q_1, q_2, q_3, &c. in the curve, from which points draw lines to B and C.

Take the $\angle q_1$ C B = an arc whose sine is $\frac{\delta}{2\,r}$, wherein δ = B q_1 = $q_1\,q_2$ = &c. and r = O B, or, because δ is, in practice, usually required to be = 1 chain, take $\angle q_1$ C B = arc whose sine is $\frac{1}{2\,r}$. The several angles may then be arranged as below.

$\angle q_1$ C B = arc to sin. $\frac{1}{2\,r}$, and $\angle q_1$ B C = $\frac{1}{2}$ $\angle$B O C $-$ $\angle q_1$ C B,

$\angle q_2$ C B = 2$\angle q_1$ C B,	$\angle q_2$ B C = $\frac{1}{2}$ $\angle$B O C $-2\angle q_1$ C B,
$\angle q_3$ C B = 3$\angle q_1$ C B,	$\angle q_3$ B C = $\frac{1}{2}$ $\angle$B O C $-3\angle q_1$ C B,
$\angle q_4$ C B = 4$\angle q_1$ C B,	$\angle q_4$ B C = $\frac{1}{2}$ $\angle$B O C $-4\angle q_1$ C B,
&c. = &c.	&c. = &c.

Therefore if theodolites be fixed at B and C, and the angles q_1 C B, q_1 B C be taken at the same time, the intersection of B q_1 and C q_1 will give the point q_1. In the same manner by taking the angles q_2 C B, q_2 B C, the intersection of B q_2 and C q_2 will give the point q_2, &c.

Ex. Let the radius O B = 80 chains, and the angle between the tangents A B, D C, when prolonged to meet, be 160°; then its supplement = $\angle$B O C = 80°, and $\frac{1}{2}$ $\angle$B O C = 40°. Also $\angle q_1$ C B = arc to sine $\frac{1}{2}$ r $(= \frac{1}{160} = .00625) = 0° \ 21' \ 29''$, whence $\angle q_1$ B C $= \frac{1}{2} \angle$B O C $- \angle q_1$ C B $= 40° - 0° \ 21' \ 29'' = 39° \ 38' \ 31''$; $\angle q_2$ C B $= 2 \ \angle q_1$ C B $= 2 \times 0° \ 21' \ 29'' = 0° \ 42' \ 58''$ and $\angle q_2$ B C $= \frac{1}{2} \angle$B O C $- 2 \angle q_1$ C B $= 40° - 0° \ 42' \ 58'' = 39° \ 17' \ 2''$, &c. The angles would be best arranged for use as follows, the opposite ones to be taken at the same time.

$\angle q_1$ C B = arc to sin. ·00625 = 0° 21′ 29″, $\angle q_1$ B C = $\frac{1}{2}\angle$B O C − $\angle q_1$ C B = 39° 38′ 31″;
$\angle q_2$ C B = 2 $\angle q_1$ C B = 0° 42′ 58″, $\angle q_2$ B C = $\frac{1}{2}\angle$B O C − 2 $\angle q_1$ C B = 39° 17′ 2″;
$\angle q_3$ C B = 3 $\angle q_1$ C B = 1° 4′ 27″, $\angle q_3$ B C = $\frac{1}{2}\angle$B O C − 3 $\angle q_1$ C B = 38° 55′ 33″;
$\angle q_4$ C B = 4 $\angle q_1$ C B = 1° 25′ 56″, $\angle q_4$ B C = $\frac{1}{2}\angle$B O C − 4 $\angle q_1$ C B = 38° 34′ 4″;
&c. = &c. = &c. &c. = &c.

This list of angles must be continued till $n \ \angle q_1$ C B can no longer be taken from $\frac{1}{2}$ $\angle$B O C, and one angle in each column being taken at the same time, as $\angle q_1$ C B at C and $\angle q_1$ B C at B will give the point q_1: and so on for the other points q_2, q_3, &c. See the following notes.

NOTE 1. — Where the obstructions are such as to prevent the stumps being put down at the consecutive points q_1, q_2, q_3, &c., it would be best to take every fourth angle, thus obtaining the points q_4, q_8, q_{12}, &c., which will be 4 chains apart on the curve, the intermediate points being left to be put in when the work of the line has progressed so far as to present a better opportunity. Or, if it should be thought preferable to avoid taking a multiplicity of angles, every fourth angle may be taken in any case, as this method is equally available whether obstructions exist or not.

NOTE 2. — *Mr. Rankine's method of setting out the curve.* As this method is a modification of the one just given, it will be proper to explain it here. It depends on the property already given, *i. e.* sin. $\angle q_1$ C B $= \frac{\text{rad. } \delta}{2 \, r}$; which angle is well known to be $= \angle q_1$ B T, or $\angle q_1$ B T = arc to sine $\frac{1}{2 \, r}$, δ being = 1 chain, and the rad. of tables = 1. A theodolite being fixed at B, and the $\angle q_1$ B T being taken, the distance δ, or 1 chain, is set off from B to q_1 in the direction of the axis of the instrument; the $\angle q_2$ B T is next taken = 2 $\angle q_1$ B T, and the distance $q_1 \, q_2 = 1$ chain applied between q_1 and the direction of the axis of the instrument, and so on to the end of the curve. It will be seen that this method, though elegant in theory, is quite as objectionable in practice as the common method, given in Problem II., since the least errors, at the commencement of the operation, will gradually multiply as the work proceeds, whether the errors be in taking the angles q_1 B T, q_2 B T, &c., or in laying out the distances B q_1, $q_1 \, q_2$, &c., or in both. Whereas in the method by two theodolites, just given, an error in taking one angle does not affect any

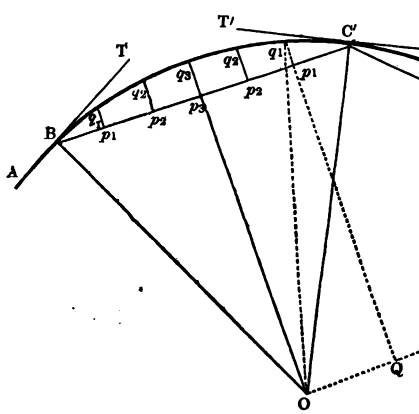
other part of the work, but simply the point to which the erroneous angle refers, the position of which point may be easily corrected. Besides, Mr. R.'s method will be found impracticable where hills, woods, buildings, &c. are within the concavity of the curve, *his method having been ostentatiously put forth as a universal method.*

Demonst. to Prob. IV.—The angles $B\,q_1\,C$, $B\,q_2\,C$, $B\,q_3\,C$, &c., being in the same segment, are well known to be equal and constant, and also equal to half the supplement of $\angle\,B\,O\,C$, and consequently the sums of the angles of the triangles $B\,q_1\,C$, $B\,q_2\,C$, &c. adjacent to $B\,C$ are equal to $\frac{1}{2}\angle\,B\,O\,C$, whence $\angle\,q_1\,B\,C = \frac{1}{2}\angle\,B\,O\,C - \angle\,q_1\,C\,B$, $\angle\,q_2\,B\,C = \frac{1}{2}\angle\,B\,O\,C - \angle\,q_2\,C\,B =$ (because equal angles stand on equal arcs) $\frac{1}{2}\angle\,B\,O\,C - 2\angle\,q_1\,C\,B$, &c. Also join $O\,q_1$ and put $B\,O = r$, and $B\,q_1 = \delta$, then it is well known that $\frac{1}{2}\angle\,B\,O\,q_1 = \angle\,q_1\,C\,B$, and, because the sides $B\,O$, $q_1\,O$ of the triangle $B\,O\,q_1$ are equal, sin. $\frac{1}{2}\angle\,B\,O\,q_1 =$ sin. $\angle\,q_1\,C\,B = \frac{\text{rad.} \times \delta}{q\,r}$; or, by taking rad. $= 1$, and $\delta = 1$ chain, as required in practice, sin. $\angle\,q_1\,C\,B = \frac{1}{2r}$, or $\angle\,q_1\,C\,B =$ arc to sine $\frac{1}{2r}$. *Q. E. D.*

PROBLEM V.

To lay out a Railway Curve by means of Ordinates or Offsets from its Chord or Chords, no material Obstruction being supposed to exist on the concave Side of the Curve to prevent the use of the Chain.

Let B C′ C be a portion, or the whole, of a curve of a railway; A T, T′ D′ and C D tangents to the curve at B, C′ and C; O the



centre, O B the radius, B C′ and C′ C chords of the curve; which chords must not exceed 40 chains, if the radius be 80 or 120 chains, but they may be 60 chains, if the radius exceed 120 (this limitation is necessary to prevent the offsets $p_1\,q_1$, $p_2\,q_2$, &c. being too long, as was observed with respect to the offsets from the tangents, in Prob. III.); and let the radius $q_3\,O$ bisect the curve and chord in q_3 and p_3

respectively. Then from the right-angled triangle B p_3 O, in which B O and A $p_3 = \frac{1}{2}$ B C′ are given, the ∠ B O p_3 = ∠ T B C may be found, which determines the position of the chord B C′; also O p_3 may be found from the same triangle being $= \sqrt{\text{O B}^2 - \text{B } p^2{}_3} = \sqrt{\text{O B}^2 - \frac{1}{4}\text{ B C}'^2}$. Put O B = r, O $p_3 = s$, and the $\frac{1}{2}$ chord B p_3 = n chains, then

$$
\begin{aligned}
q_1\, p_1 &= \sqrt{r^2 - (n-1)^2} - s^*, \\
q_2\, p_2 &= \sqrt{r^2 - (n-2)^2} - s, \\
\&\text{c.} &= \quad \&\text{c.} \\
q_3\, p_3 & \quad \sqrt{r^2 - (n-3)^2} - s = r - s
\end{aligned}
$$

$q_3\, p_3$ being supposed to be the offset at the middle point of the curve B C′, not the third offset as shown in the figure, it being impossible to draw all the offsets without confusing the figure or making it unnecessarily large. After reaching the middle point q_3 of the curve the same offsets are repeated in an inverted order till the curve shall have been set out to C′. The same operation may be repeated as often as necessary, till the whole curve be completed, observing to make the ∠B C′ C = 2 complement of ∠ T B C′, which has been already found. See the following Notes.

NOTE 1.—When B C′ is the whole curve, and its chord Bp_3C includes a fractional part of a chain, the distance of the offsets on each side of the middle of the curve will be less than one chain; therefore that distance, and consequently the following ones, must be made equal, as was shown with respect to the distances in Prob. III.

NOTE 2.—If the last chord, which suppose to be C′ C, be less than the preceding chord or chords, the ∠ O C′ C must be found, and added to the ∠ O B C′ or O C′ B, which will give the ∠ B C′ C, showing the direction of the chord C′ C.

NOTE 3.—This method of laying out the curve is seldom used, on account of the calculations it involves. It may, however, be used with advantage where a winding river or cliff is close to the convex side of the curve, or protrudes in some places a little through the curve; thus preventing the use of any other method, except that in Prob. IV., which requires two theodolites.

Definition of the Compound Curve. (See figures to Prob. VI.)

The compound curve B C C′ C″, joining the tangents A B, D C″, is composed of three circular arcs B C, C C′, C′ C″, having common normals O C, O′ C′ at their points of junction C, C′; and therefore

* *Demonstration.*—Draw O Q parallel to B C′, q_1Q ⊥ to O Q, and join q_1O; and let n = number of chains in $\frac{1}{2}$B C′ or p_3C′, and C′p_1 = 1 chain. Then $p_3 p_1$ = O Q = $n - 1$; and (Euc. I. 47.) q_1 Q = $\sqrt{r^2 - (n-1)^2}$; whence $q_1 p_1 = \sqrt{r^2 - (n^2 - 1)^2} - p_1$Q: but p_1Q = p_3O = s, ∴ $q_1 p_1 = \sqrt{r^2 - (n-1)^2} - s$. Similarly $q_2 p_2$ is found $= \sqrt{r^2 - (n^2 - 2)^2} - s$, &c. *Q. E. D.*

common tangents at the same points, the radii of the three portions of the curve being respectively O B = O C, O′ C = O′ C′ and O″ C′ = O″ C″. This kind of curve is adopted where the line is required to pass through given points, as C and C′, to avoid obstructions, or where a principal station or *terminus* is at or near C″; in the latter case the radius O″ C″ may, if required, be less than 80 chains.

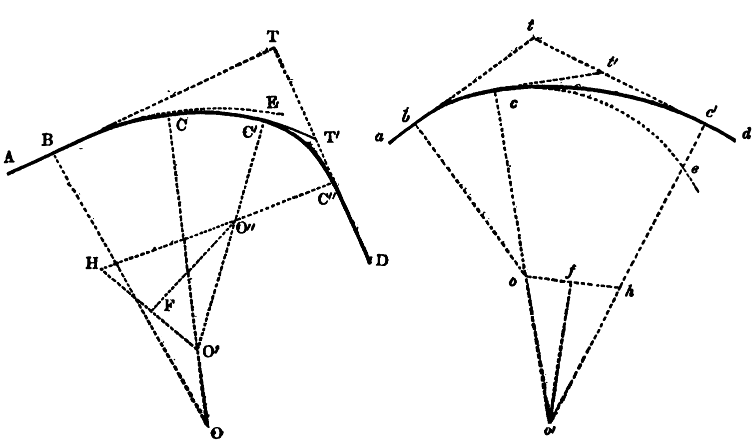
The compound curve may consist of two, three, or more portions of different arcs; thus the curve *b c c′* consists of two portions, *b c, c c′*.

PROBLEM VI.

1. *To find the several radii of the Compound Curve mechanically.*

Let A B, D C″ be the tangental portions of the line, which are required to be joined by a curve passing through the points C, C′, *the point* C″ *not being given.* Select a curve-ruler such that being applied to touch A B at B, it may also pass through C: if this curve do not pass through C′, but through some other point E, another curve-ruler of less radius, in this case, must be selected, and such that it may touch the arc B C at C without cutting it or its prolongation towards E, and also pass through the point C′: if this curve cut the prolongation C″ T′ of the tangent D C″, another curve-ruler of less radius than the last one must be selected, and such that it may touch curve C C′ at C′ and the tangent D C″ at C″: thus completing the curve B C C′ C″. The radii O C, O′ C′, O″ C″ may be determined from the curve-rulers, as in Case I. Prob. I.

2. *To find the radius c′ o′ of the Compound Curve b c c′ geometrically, the starting point b and the radius b o being given.*



From the given point b in the tangent $a\,b$, draw the given radius $b\,o \perp$ to $a\,b$; and draw the curve to some point c, where it is found convenient to change the radius: draw the radius $o\,c$, and $\perp$ thereto draw $c\,t'$, meeting the tangent $d\,t$ in t'; make $t'\,c' = t'\,c$, and from c' draw $c'\,o' \perp$ to $t\,c'$ meeting $c\,o$ prolonged, if necessary, in o'; then o' is the centre of the arc $c\,c'$ of the curve, conformable to the nature of tangents.

The method of constructing the curve, when it consists of three or more parts, is sufficiently obvious.

3. *One of the two radii of the Compound Curve, and its starting and closing points being given, to find the other radius.*

Let $a\,b$, $c'\,d$ be the tangents, b and c' the starting and closing points of the curve. Draw the perpendiculars $b\,o = c'\,h =$ given radius to the tangents; join $o\,h$, and bisect it in f; draw $f\,o' \perp$ to $o\,h$, meeting $c'\,h$ prolonged in o'; join $o'\,o$ and prolong it till $o\,c = c'\,h$: then the points o, o', are the centres of the arcs $b\,c$, $c\,c'$, which constitute the compound curve, $o'c = o'\,c'$ being the radius required.*

NOTE. — In the compound curve B C C′ C″, where the radius C″ O″, which is to be found, is less than the preceding radius C O, the $\perp$ C″ H is made = C O; H O is joined and bisected in F; and F O″ drawn $\perp$ to H O′, meeting C″ H in O″, which is the centre of the arc C′ C″, &c.

Definition of the Serpentine or S *Curve.*

The serpentine curve B G C is used in railways, when obstructions or some other cause render its adoption preferable; it consists of two circular arcs of different or the same radii, having their convex sides turned in opposite directions, like the letter S, whence it is sometimes called the S Curve; the two portions B G, G C of the curve have a common normal O G O′ at their point of junction G, and therefore a common tangent at the same point. This curve affords the most easy means of joining two parallel, or nearly parallel, portions of a line of railway.

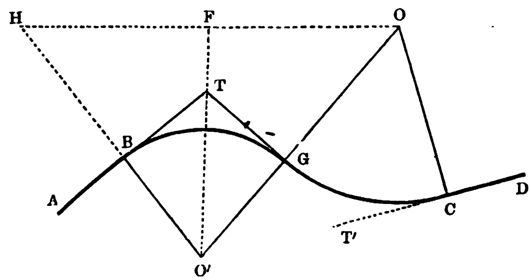
PROBLEM VII.

1. *When one radius and its tangental point are given, to find the other radius and tangental point of the serpentine curve geometrically.*

From the given tangental point C draw the given radius C O $\perp$ to the tangent C D, and draw the curve C G to some point G,

* *Demonstration.* — Since $o\,f = f\,h$, and $f\,o'$ is $\perp$ to $o\,h$, $o\,o' = o'\,h$, also $b\,o$ was made $= c'\,h = o\,c$; $\therefore o'\,c = o'\,c'$, and the normal $o'\,o\,c$ is common to the arcs of the curve. *Q. E. D.*

where it is found convenient that it should have its point of contrary flexure: through O G draw the normal O G O′; from



G draw G T ⊥ to O G O′ to meet the tangent A T; make T B = T G; and draw B O′ ⊥ to A T, meeting O G O′ in O′; then O′ is the centre, and O′ B = O G is the radius of the curve B G, as is evident from the nature of tangents.

NOTE. — The radius O′ B, and tangental point B may be found mechanically, *i. e.* by the curve-rulers, as in Problems I. and VI.

2. *When the tangental points and one of the radii of the Serpentine Curve are given, to find the other radius geometrically.*

From the given tangental points C and B draw C O, B H, repectively ⊥ to the tangents C D and B A, and equal to the given radius; join O H, and bisect it in F; draw F O′ ⊥ to O H, meeting H B prolonged in O′, and join O, O′; making O′ G = O′ B; then O′ is the centre, and O′ B = O′ G is the radius of the portion B G of the curve, as required.*

NOTE 1. — The radius B O′ may be found mechanically as in the preceding case.

NOTE 2. — The radius O′ B may be found from the following formula, wherein δ = B C, r = given radius O C, $\alpha = \angle$ T B C and $\alpha' = \angle$ T′ B C.

$$O'B = \frac{\delta(\delta - 2r \sin.\, \alpha')}{2\left(\delta \sin.\, \alpha + 2r \sin.^2 \frac{\alpha - \alpha'}{2}\right)}.$$ See investigation and fig. to Prob. VIII.

Ex. Let $\delta = 200$ chains, $r = 110$ chains, $\alpha = 45°$, and $\alpha' = 20°$; then sin. $\alpha = ·70711$, sin. $\alpha' = ·34202$, $\frac{\alpha - \alpha'}{2} = \frac{45° - 20°}{2} = 12°\ 30'$, the sine of which is ·21644; and by the formula $O'B = \frac{200(200 - 220 \times ·34202)}{2(200 \times ·70711 + 220 \times ·21644^2)} = \frac{200 \times 124·7556}{2 \times 151·7282} = 82·22$ chains.

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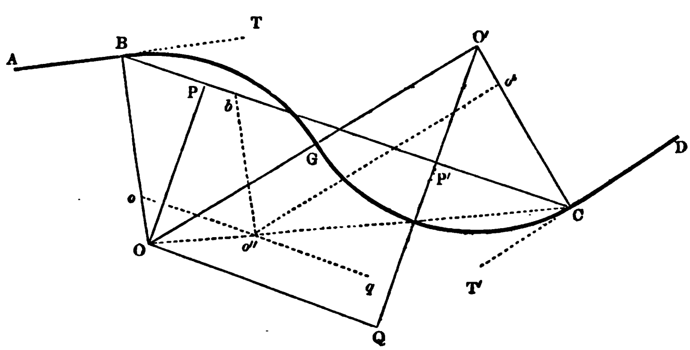
* The *demonstration* in this case is similar to the one given to Prob. VI.

It may thus be readily ascertained whether the required radius O′ B is greater, equal to, or less than 80 chains; if less, the given radius r ought either to be diminished, or the distance B C of the tangental points ought to be increased according to circumstances, in order that O′ B may be of the required length, assuming that the portion B G of the curve is not near a *terminus* or principal station.

PROBLEM VIII.

1. *When the two portions of the Serpentine Curve have the same radius, to determine that radius geometrically, the tangental points and their distance being given.*

Let A B, D C, be the tangents, B and C the given tangental points, and B C the given distance. Draw B o = C o' respectively ⊥ to



A B, D C, and of any convenient length; through o, parallel to B C, draw $o\,q$ indefinitely; with the compasses apply $o'\,o'' = 2\,C\,o' = 2\,B\,o$; through C, o'' draw C o'' O, meeting B O prolonged in O; and through O, parallel to $o''\,o'$, draw O O′, meeting C o' prolonged in O′; then O and O′ are the centres, and O B and O′ C are the equal radii of the serpentine curve B G C, the common normal of the portions B G, G C of the curve, being O G O′ = 2 B O = 2 C O′.*

2. *To find the common radius of the two portions of the Serpentine Curve by calculation, the same things being given as in the preceding case, and the angles* T B C, T′ C B.

Put B C = δ, B O = C O′ = r, ∠ T B C = α, and ∠ T′ C B = α';

* *Demonstration.* — Draw $o''\,b$ parallel to O B; then by similar triangles,

$$C\,o' : o'\,o'' = 2\,C\,o' :: C\,O' : 2\,C\,O' = O\,O',$$

$$\text{and } C\,o'' : C\,O :: C\,o' = B\,o = b\,o'' : C\,O'$$

$$b\,o'' : B\,O = C\,O'. \quad \textit{Q. E. D.}$$

then $r = \dfrac{\delta}{\text{sin. } \alpha + \text{sin. } \alpha' + 2 \text{ sin. of arc to cos. } \frac{1}{2} (\text{cos. } \alpha + \text{cos. } \alpha')}$.*

Ex. Let $\delta = 200$ chains, $\alpha = 27°$, $\alpha' = 50°$: then from a table of nat. sines sin $\alpha = .45399$, its cos. $= .89101$; sin. $\alpha' = .76604$, its cos. $= .64279$: whence $\frac{1}{2}$ (cos. α + cos. α') $= \frac{1}{2}$ (.89101 + .64279) $= .76690$ = cos. and 2 sin. of arc to cos. .76690 is 1.28334; therefore $r = \dfrac{200}{.45399 + .76604 + 1.28334} = \dfrac{200}{2.50337} = 79.89$ chains, or nearly 80 chains, the radius required.

NOTE. — The method of forming the serpentine curve with a common radius is much to be preferred to any other, when the nature of the ground will admit of its being done; and more especially so, when the *data*, as in the preceding example, will only just give a common radius of 80 chains, whereas, if the radius of one ot the portions of the curve had been taken greater than 80 chains, the other radius would have necessarily been less than 80 chains.

PROBLEM IX.

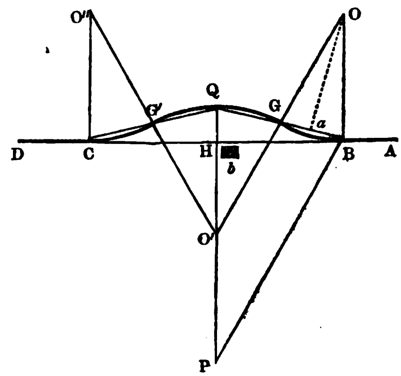
To make a given deviation H Q *from a straight portion of a Line of Railway,* A H D, *by means of three Curves,* B G, G Q G′, G′ C, *having their radii,* O B, O′ Q, O″ C, *all equal; in order that the lateral Works of the Line may avoid the Building, or other Obstruction,* b, *which is close to the centre of the straight portion of the Line.*

* *Investigation.* — Draw P O, P′ O′ ⊥ to B C; through O draw O Q parallel to B C, meeting O′ P′ prolonged in Q: then O O′ = 2 r, B P = r sin. α, O P = P′ Q = r cos. α, C P′ = r sin. α', O′ P′ = r cos. α', O′ Q = O′ P′ + P′ Q = r (cos. α + cos. α'): whence, from the right-angled triangle O Q O′, sin. ∠ O′ O Q = $\frac{1}{2}$ (cos. α + cos. α'), which is therefore given, whence the comp. of ∠ O′ O Q = ∠ O O′ Q is known, and may be thus expressed, sin. ∠ OO′ Q = sin. of arc to cos. $\frac{1}{2}$(cos. α + cos. α'). Whence O Q = P P′ = O O′ × sin. ∠ O O′ Q = 2 r × sin. of arc to cos. $\frac{1}{2}$(cos. α + cos. α'), δ = B C = B P + P P′ + P′ C = r sin. α + r sin. α' + 2 r sin. of arc to cos. $\frac{1}{2}$(cos. α + cos. α'), from which

$$r = \frac{\delta}{\text{sin. } \alpha + \text{sin. } \alpha' + 2 \text{ sin. of arc to cos. } \frac{1}{2}(\text{cos. } \alpha + \text{cos. } \alpha')}. \quad Q.\ E.\ I.$$

When the radii are unequal, and one of them, as r = O′ C is given, and the other R = B O is required; then O′ O = R + r, B P = R sin. α, P O = P′ Q = R cos. α, O′ Q = R cos. α + r cos. α', and O Q = P P′ = $\sqrt{O'O^2 - O'Q^2} = \sqrt{(R+r)^2 - (R \cos. \alpha + r \cos. \alpha')^2}$, $\delta = R \sin. \alpha + r \cos. \alpha' + \sqrt{(R+r)^2 - (R \cos. \alpha + r \cos. \alpha')^2}$. By transposing and squaring, and remembering that sin.^{2} + cos.^{2} = 1, &c. there results $\delta^2 - 2\delta(R \sin. \alpha + r \sin. \alpha') = 2 R r (1 - \cos. \alpha \cos. \alpha' - \sin. \alpha \sin. \alpha') = 2 R r (1 - \cos. \overline{\alpha - \alpha'}) = 4 R r \sin.^2 \frac{\alpha - \alpha'}{2}$, whence $R = \dfrac{\delta(\delta - 2 r \sin. \alpha')}{2\left(\delta + 2 r \sin.^2 \frac{\alpha - \alpha'}{2}\right)}$. This formula is used for finding the value of B O′, Note 2., Prob. VII., where the symbols are defined. *Q. E. I.*

1. *Construction.* From the given point H draw $HQ =$ given deviation $\perp$ to AD; on QH prolonged, take $QO' = OP =$ given radius; with the compasses apply $PB = QP =$ twice given radius; draw $BO \perp$ to AD; through O′ draw O′GO parallel to PB, meeting OB in O; make $CH = HB$; and join CQ, QB, the latter cutting OO′ in G, and the former cutting O′O″ (which is similarly drawn to OO′) in G′: then B and C are the starting and closing points of the curve, of which the separate portions are BG, GQG′, G′C, and the chords BG, GQ, QG′, G′C are all equal.*



2. *Calculation.* Take $BH = HC = \sqrt{QH(4BO - QH)}$, and $BG = GQ = QG' = G'C = \sqrt{BO.QH}$, which chords of the arcs BG, GQ, &c. thus become known; and, since the common radius BO is given, the construction of the curve is obvious.†

Ex. Let the given deviation $QH = 2$ chains, and the common radius $BO = 85$ chains; then $BH = HC = \sqrt{2(340-2)} = \sqrt{676} = 26$ chains, and $BG = GQ =$ &c. $= \sqrt{2 \times 85} = \sqrt{170} = 13.04$ chains.

REMARKS ON LAYING OUT THE CURVES IN THE FOUR LAST PROBLEMS.

Having in the four last problems given various methods of determining the radii and common normals, indicating the positions of the tangent points of the parts of the compound, serpentine, and deviation curves, the method of laying out the curves themselves by Problems II., III., IV., or V., according to circumstances, will be readily seen, recollecting that when junction-points of curves of

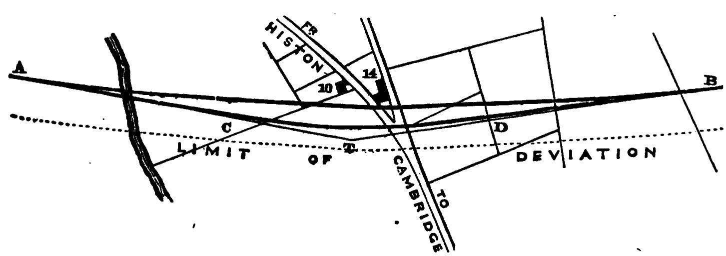
* *Demonstration.* — Because QP, BO are parallel, as are also O′O, PB; $O'P = BO =$ (by const.) $QO' =$ given radius, and $O'O = PB = 2QO' = 2GO'$; $\therefore GO' = GO = QO = BO$, which are similarly proved to be $= G'O' = G'O'' = CO''$. *Q. E. D.*

† *Demonstration.* — Draw Oa perpendicular to BQ, bisecting it in a; then by similar triangles $BO : Ba = \frac{1}{2}BQ :: BQ : QH$, whence $BQ^2 = 4BO.QH$, or $BG^2 = \frac{1}{4}BQ^2 = BO.QH$, or $BG = \sqrt{BO.QH}$. Also $BH^2 = HC^2 = BQ^2 - QH^2 = 4BO.QH - QH^2 = QH(4BO - QH)$, or $BH = HC = \sqrt{QH(4BO - QH)}$. *Q. E. D.*

different radii occur, as C C′, first fig. to Prob. VI., to commence the operation afresh, by using the radii and tangents of the respective portions of the curve.

Examples showing how expensive Severance of Property, &c., is unnecessarily made by improperly laying out Railway Curves.

It has been shown in Prob. I. that the curve to be adopted, in joining two straight portions of a line of railway, is that which avoids, as much as possible, expensive severance, cuttings, bridges, &c., provided its radius is within legal limits. The following examples (which came immediately under the author's observation) will show some very ignorant violations of this rule.



In the annexed figure, A B is a curve of three miles radius in the Wisbeach, St. Ives, and Cambridge Railway, A and B being the tangent points, A T, T B, the tangents prolonged to meet at T, which point is within the limit of deviation. It will be seen that the curve A B passes through two gardens, Nos. 10. and 14. (the numbers being those on the railway-map); the boundary of the former is every where within three chains of the line; the railway company are, therefore, liable to purchase the whole of No. 10. In the latter, *i. e.* No. 14., the line passes through the garden, close to three cottages, and crosses two roads, one on each side of the garden. The cottages will therefore be required to be pulled down, besides the expensive severance of the garden, and two pairs of gates for the roads to be constantly attended, or the two roads diverted into one, for the railway passes on the level of the road, or nearly so. Now, as the ground is perfectly level within the limits of deviation, it will be readily seen that by prolonging the straight parts of the line from A to C and from B to D, and by substituting the curve C D of one mile radius, instead of the curve A B, the expensive severance in Nos. 10. and 14. would have been avoided, and only one road required to be crossed.

2. An expensive severance, including bridges, &c., was also made unnecessarily in the same line, at Westwick, about two miles westward of the one just referred to, by laying out the line through a gentleman's park, within 100 yards of his house, and crossing and recrossing a brook, 16 feet in width, within the space of a few chains, leaving the brook, between the points of crossing, only about one chain from the line in the widest part. The line where this severance was made being a curve of $1\frac{1}{2}$ mile radius, at about the middle of the curve, which might have been avoided by substituting a curve of one mile radius, and changing the position of one of the tangents about 10 links at the commencement of the curve, the limits of deviation, and the ground, (being almost perfectly level,) admitting of such a change in the line; by which two oblique bridges for crossing and recrossing the brook would have been rendered unnecessary, as well as the expensive severance.

I presume the head engineer of the line would cause these expensive blunders, thus ignorantly made, to be rectified previous to the construction of the line: but these matters are often strangely overlooked, or disregarded in the hurry of railway practice.

REMARKS ON THE INVENTION OF THE FOUR PRECEDING METHODS OF LAYING OUT RAILWAY CURVES ON THE GROUND, AND ON THE METHODS PUBLISHED BY OTHER AUTHORS.

As seven or eight gentlemen, during the last and present years (1846 and 1847), have published on the subject of laying out railway curves on the ground, no doubt with a view to enlighten the railway world, their methods, at least the practically useful ones, not essentially differing from the four methods which I invented about a quarter of a century ago, and afterwards published in the Gentleman's Diary, being adapted to all cases that can occur, I therefore think it right to claim the invention. My attention having been drawn to this subject when the first portion of the Stockton and Darlington Railway was laid out, and being then myself accustomed to mathematical investigations, and a land surveyor residing near the above named railway, and acquainted with the surveyors employed in laying it out, I communicated three of my methods to them; some of whom were afterwards employed on the Liverpool and Manchester Railway, as well as on various other railways in different parts of the kingdom. I at the same time communicated these methods to several of my scientific friends, many of whom are still living, and can be referred to, if required; among whom was Professor Leybourn, of the Royal M. College, Sandhurst, to whom I sent them, with a fourth method (see Prob. IV.), in 1824, for insertion in the Gentleman's Diary, or his Math. Repository, to the former of which I had previously contributed on mathematical subjects for

some years. But partly on account of the length of my paper excluding the claims of other contributors, and partly *on account of the very little importance then attached to railways*, I could not prevail on Professor Leybourn to insert my paper for several years, nor did I then myself attach so much importance to it as it now appears to deserve, not thinking that railways would become so general, as they now are, in my own time. In 1834, I saw Professor Leybourn in London, who, as railways had then begun to assume considerable importance, promised to insert my paper (of which I gave him an improved copy) at his earliest convenience, either in the Gentleman's Diary or in his Mathematical Repository: he also, in conjunction with Dr. Gregory, of Woolwich, about the same time, recommended me as assistant to Mr. Vignolles, a well known railway engineer, in which position I continued till the surveys commenced under the Tithe Commutation Act, in which I was several years engaged for first-class maps, having surveyed seven parishes in Sussex, *the map of one of which* (Tillington), *on account of trifling informalities in the Field-Book, was tested on the ground, and found unexceptionably correct, a rare occurrence among the numerous maps of other surveyors, which were tested about the same time.* I mention this, which may seem apart from the subject, to show that I am not only a projector of Mathematical theories, but can also execute them practically with the greatest accuracy.

My four methods of laying out railway curves, having stood over till the claims of other contributors were satisfied, were at length inserted in November, 1837, in the Gentleman's Diary for the following year; these methods, on account of my connection with engineers, surveyors, and scientific men, having been communicated by me to at least fifty persons, some of whom were my private pupils, between the time of their invention and their publication.

The first method which I invented, was that given in Problem II., page 372.; it was eagerly adopted by railway engineers and surveyors (who at that time knew little or nothing of the Mathematics), because of its involving very little calculation, and not requiring the use of an angular instrument. This method, afterwards known by the name of the *common method*, has been used in laying out the curves of nearly all the principal railways in this kingdom, as well as those in foreign countries. This method, notwithstanding its extensive use, is defective in practice, on account of its requiring the coupling together of so many short lines to one another, since very small errors made at the commencement of the curve will produce a great deviation at its termination, especially if it be a long one, and the ground be rough or uneven; so that the curve has frequently to be retraced from three to seven or eight times, before it can be got right, excepting where the positions of one of the tangents can be changed to adapt it to the curve, which is not commonly the case.

In consequence of the defect of the above method, I prepared the method (given in Problem III.) of laying out the curve by ordinates or offsets from its tangents, or from a series of tangents, according to the length of the curve. This method I always considered preferable to any other, especially where the curve is a long one; and it is now generally adopted by the judicious portion of engineers and surveyors, the curve by this method being readily laid out in its true position. But this method cannot always be adopted in practice on account of obstructions, such as buildings, woods, cliffs, rivers, &c., on the convex side of the curve.

To remedy this inconvenience I produced a third method (given in Problem V.) of laying out the curve from its chord, or from a series of chords, no obstructions

being supposed to exist on the concave side of the curve. This method is seldom used on account of the complex calculations required in finding the offsets, &c.

I also invented, about the same time, a fourth method (given in Problem IV.) of laying out the curve by two theodolites, on ground where the use of the chain is prevented by swamps, mosses, pits, quarries, &c.

With respect to the gentlemen who have lately published (in 1845 to 1847) on the subject of laying out railway curves, Mr. Law (who publishes in conjunction with Mr. F. W. Simms) recommends three of my methods, with what he, doubtless, considers to be improvements on them, by taking four-chain chords, &c., in my common method, with various formulæ, without investigation, to determine minutiæ unappreciable in almost every case occurring in practice, which will be any thing but welcome to the majority of practical men, and will not remedy the defects of the method. In the method of laying out the curve from its tangents, he recommends its centre to be found, in order that the offsets may be laid out radially, which is in most cases impracticable, and in every case unnecessary; indeed, I never heard of its being done. He also recommends Mr. Rankine's method, which is even more defective than my common method. (See Problem IV., note 2.)

Mr. Castle recommends the method by curve-frames, which is theoretically but a modification of my common method, and equally defective: hence, five pages of his work are devoted to explaining the method of making his defective curves fit one another and touch their tangents.

Mr. W. Hill recommends Mr. Rankine's method, above referred to, and my method, given in Problem IV., also an "original formula" for finding the equal radii of the serpentine curve; which formula, judging from his circuitous investigation of it, is, I think, doubtless, "original" to him: but the same formula, with the geometrical construction of the curve, was placed in professor Leybourn's hands for publication many years ago. (See Problem VIII.)

None of the authors, above referred to, either claim the invention of the methods they have published, or acknowledge from whence they are derived, excepting what they have taken from Mr. Rankine, as already stated. Others, who have published on the same subject, either lately or some years past, in periodicals, have only produced isolated methods, which are either mere copies of one or other of my methods, or such as will never be adopted in practice.

I do not mean to infer, from what I have already said, that some other person or persons would not have invented the same methods of laying out curves, provided I had never invented them; as the subject involves no great difficulties, in a mathematical point of view, the difficulties being chiefly of a practical character, to surmount which a combination of the knowledge of those practical difficulties, of the use of surveying instruments, and of Geometry and analytical Trigonometry is requisite.

In concluding these remarks I have the consolation to say, *that I completely anticipated, a quarter of a century ago, all that has since been done by others on this subject, that can be considered of real practical utility; and of all the inventions connected with railways, my methods of laying out curves on the ground, especially two of them, have been the most generally adopted in practice, the other two methods being only required in particular cases.*

T. BAKER.

SECTION II.

CUTTING AND EMBANKING RAILWAYS.

On Setting out the Width of Ground for a Railway.

AFTER the centre stumps of the railway have been put down, which, as before observed, are usually at the distance of one chain, the line must next be carefully levelled, and the number of the stumps entered in the Level-Book, in a vertical column; and opposite each number in a second column, the depth of the cuttings or embankments (see Level-Book, page 39); and in a third column, the computed, or horizontal half-width of the surface cuttings, as found by Problems I. and II. following; the depths of the cuttings and embankments being estimated from the balance line, which is 2 feet below the line of the rails or gradients, the 2 feet being filled up with gravel to form the way, and the beds for the sleepers of the rails.

The side-stumps are next to be put down, which must be placed two on each side of every centre stump, in a direction perpendicular to the length of the line; or, if the line be curved, in a direction perpendicular to the tangent to the curve at the centre stump: the two interior stumps, *i. e.* those next to the centre one, to mark the width of the cuttings, and the two exterior ones to mark the ditches of the side fences. The distances of every two of the interior stumps are to be entered in the Level-Book, opposite the number of the centre stump, in the main section, in order to ascertain the quantity of cuttings for the contractor; and the distances of every two of exterior stumps from the centre one, are to be similarly entered, to ascertain the quantity of land which will be required from each proprietor for the works of the line; which last may be calculated accurately by the method of equi-distant ordinates. (See Prob. VI.)

PROBLEM I.

To set out the Width when the Surface of the Ground is laterally on the same Level as the intended Railway.

FROM the centre stump, perpendicular to the direction of the line, set out half the bottom-width for the cutting, to which add the width of the side-fence, putting down a stump at each distance; then repeat the operation on the other side of the line.

Ex. If the bottom-width of the railway be 30 feet, and the breadth of one of the side fences be 12 feet, required the widths for cutting and for fences.

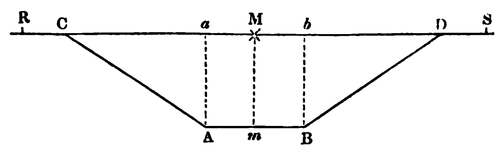
$30 \div 2 = 15$ ft. = dist. of side-stump from centre for cutting.

$15 + 12 = 27$ ft. = dist. of side-stump from centre for fence.

Therefore, $15 \times 2 = 30 =$ whole width for cutting.
and $27 \times 2 = 54 =$ whole width for fences.

PROBLEM II.

To set out the Width of Cuttings or Embankments, when the Surface of the Ground is laterally Level, and at a given Height above the Level of the intended Railway, the Ratio of the Slopes being given.



Let ABCD be a cross section of the cuttings, R S the horizontal surface of the ground, A B the bottom-width, A C, B D, the slopes, $Mm = Aa = Bb$ the perpendicular depth, and M the middle stump. Multiply the given depth Mm by the ratio of the slopes, to which add half the bottom-width Am, or aM: set out this distance from M to C for the half-width of the cuttings, to which add the width of the side fence for the whole half-width, repeating the same operation on the other side of M.

Ex. If the bottom-width A B or ab be 30 feet, the depth of the cuttings 28 feet, and the ratio of the slopes $1\frac{1}{2} : 1$, and the width of one of the side fences 12 feet, required the width of the cuttings, and of the land for the works of the railway.

$28 \times 1\frac{1}{2} + \frac{30}{2} = 42 + 15 = 57$ feet $= MC = MD = \frac{1}{2}$ width of cuttings; and $57 + 12 = 69$ feet $= MR = MS = \frac{1}{2}$ width of land. The doubles of which are the whole widths.

If $w =$ bottom-width $=$ AB, $a =$ depth of cuttings $= Mm$, $f =$ width of one of the side fences $=$ RC, and the ratio of the slopes $r : 1$. Then

$$1 : r :: a : aC = ar, \text{ hence } ar + \tfrac{1}{2}w = MC = MD,$$
$$ar + \tfrac{1}{2}w + f = MR = MS,$$
$$\text{and } 2\left(ar + \tfrac{1}{2}w + f\right) = w + 2ar + 2f = RS.$$

Construction. — Draw A B = given width = 30 feet, perpendicular to which, at its middle point m, draw m M = given depth = 28 feet; through M parallel to A B draw R M S; through A and B draw A a, B b parallel to M m; make a C = b D = $1\frac{1}{2}$ times M m, join A C, B D,

and make R C, D S = width of one of the side-fences = 12 feet. Then A B D C is a cross-section of the cuttings, C D the surface-width thereof, and R S the whole surface required for the railway.

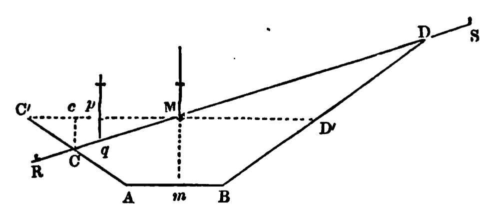
By inverting the last figure, so that A B D C may represent the cross-section of an embankment, it will be readily seen that the same method will apply, for setting out its half-widths, M C, M R, as that just given for the cuttings; as the dimensions are the same in both cases.

NOTE 1. —The ratio of the slopes is the proportion of the batter C a to the depth A a or M m. Thus, when the ratio is 1 : 1, C a = A a; when the ratio is 2 : 1, C a = 2 A a, &c. This ratio depends on the nature of the material through which the cuttings are made; if it is close jointed hard rock, the ratio is $\frac{1}{4}$ or $\frac{1}{2}$ to 1; if soft, or loose jointed rock, or strong clay, the ratio is 1 or $1\frac{1}{2}$ to 1; if springy ground, or loose sand, the ratio is 2 or $2\frac{1}{2}$ to 1.

NOTE 2. — The computed half-widths, in the third column of the Level-Book, are found by this Problem.

PROBLEM III.

To set out the Width of the Cuttings, when the Surface of the Ground is laterally-sloping, the Height of the Centre Stump above the Level of the intended Railway, the Ratio of the Slopes, and the lateral Fall or Rise of the Ground in a given horizontal Distance being given.



Let A B D C be a cross-section of the cuttings, R S the sloping surface of the ground, AB the bottom-width, M m the depth of the cuttings, M the middle stump, A C, B D, the slopes, and C′ D′ a horizontal line passing through M, M C′ = M D′ being the computed half widths.

Having set the level so that by turning the telescope two or three chains length of the line may be seen, if possible, on both sides of it, place a levelling staff at M, and another at q, and observe the level readings on them, the difference of which will be p q; measure with the tape-line in feet the distance M q on the slope, and the horizontal distance M p; take the computed half-width from the Level-Book, or find it by Prob. II., and multiply it by the distance M q on the slope, and reserve the product. Add and subtract the product of the difference of the stave readings, and the ratio of the slopes, to and from the horizontal distance M p, and reserve the sum and difference; divide the reserved product by the reserved sum for the corrected

half-width M C, and by the reserved difference for the corrected half-width M D.

Ex. Let the depth of the cuttings at the centre-stump M be 20 feet, the bottom width A B = 30 feet, the distance M q on the slope = 25 feet, the distance M p on the level = 24 feet, the difference of the stave-readings $p\,q$ = 7 feet, and the ratio of the slopes $1\frac{1}{2} : 1$; required the corrected half-widths M C, M D.

$$20 \times 1\tfrac{1}{2} + \frac{30}{2} = 45 \text{ feet} = \text{computed half-width.}$$

$$\begin{array}{r} 45 \\ 25 \\ \hline 1125 \end{array} \text{ reserved product.}$$

$$\begin{array}{r} 24 \\ 7 \times 1\tfrac{1}{2} = 10\tfrac{1}{2} \\ \hline \end{array}$$

reserved sum $34\frac{1}{2})1125(32.6$ feet = cor. $\frac{1}{2}$ width M C.

reserved diff. $13\frac{1}{2})1125(83.33$ feet = cor. $\frac{1}{2}$ width M D.

Put M C′ = b, $p\,q = h$, M $q = s$, M $p = l$, M C = x, M D = x', and the ratio of the slopes $r : 1$.

$$\text{Then } x = \frac{b\,s}{l + r\,h} = \text{M C*},$$

$$\text{and } x' = \frac{b\,s}{l - r\,h} = \text{M D}.$$

If the numbers in the preceding example be substituted in these formulæ, the work will stand thus:

$$\text{M C} = \frac{b\,s}{l + r\,h} = \frac{45 \times 25}{24 + 7 \times 1\frac{1}{2}} = \frac{1125}{34\frac{1}{2}} = \frac{2250}{69} = 32.6 \text{ feet,}$$

$$\text{M D} = \frac{b\,s}{b - r\,h} = \frac{45 \times 25}{24 - 7 \times 1\frac{1}{2}} = \frac{1125}{13\frac{1}{2}} = \frac{2250}{27} = 83.33 \text{ feet;}$$

* *Demonstration.* Draw C c ⊥ to C′ D′; then by similar triangles, $s : h :: x : \frac{h\,x}{s}$ = C c, hence C′ $c = \frac{r\,h\,x}{s}$, and M c = C′ M − C′ $c = b - \frac{r\,h\,x}{s}$; again, by similar triangles, $l : h :: b - \frac{r\,h\,x}{s}$: C $c = h\frac{(b\,s - r\,h\,s)}{l\,s} = \frac{h\,x}{s}$, whence $x = \frac{b\,s}{l - r\,h}$ = M C; and in a similar manner is found $x' = \frac{b\,s}{l - r\,h}$ = M D. *Q. E. D.*

Cor. When the difference of the level-readings is so large that the horizontal distance l cannot be conveniently measured, the value of $l = \sqrt{s^2 - h^2}$ must be substituted, thus giving

$$\text{M C} = \frac{b\,s}{\sqrt{s^2 - h^2} + r\,h}, \qquad \text{and M D} - \frac{b\,s}{\sqrt{s^2 - h^2} - r\,h}.$$

to each of which values of the corrected half-width, the width of one of the side fences must be added for the whole breadth of land required.

NOTE 1. — When the difference of level-readings is small, l may be taken $= s$ without material error.

NOTE 2. — A little practice will make this method of finding the corrected half-widths very easy. It is evidently much superior to the method of finding the same things by the blundering approximations now in use, especially where the difference of the levels of C and D is very considerable, and the surface of the ground has a regular slope, which often happens to be the case, or very nearly so; or, if the surface be a little uneven, a skilful engineer, by applying the tape-line in the general direction of the surface of the ground, will be readily able to correct the difference $p\,q$ of the level-readings, so as to adapt it to his purpose. If, however, the surface of the ground be very uneven, which sometimes happens to be the case, it will be best to use the method of approximation, as in Prob. V.

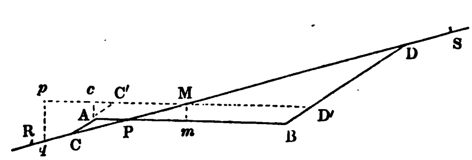
Construction. Draw the cross-section A B D′ C′ as in the preceding Problem; make M p=given horizontal distance; draw $p\,q$ perpendicular to C′ D′ and equal to the difference of the level readings; and through M q draw R S, cutting A C′ in C and B D′ prolonged in D: then A B D C is the required cross-section of the cuttings, R C, D S being the breadths of the side fences.

By reversing the cross-section, it will be seen that the preceding calculation and construction will also apply to an embankment; but in this case the lesser distance, M C, must be measured up the slope, and the greater, M D, down it.

PROBLEM IV.

To set out the Widths when the Surface of the Ground is laterally sloping, and when the Cross-section of the Works consists partly of a Cutting and partly of an Embankment, the things given being the same as in the preceding Problem.

Let A C P B D be a cross-section of the works of a railway, consisting partly of the cutting B P D, and partly of the embankment A P C; A B the bottom-width; M the centre stump, and M m the depth of the cutting;



R S the sloping surface of the ground, &c. When the embanked part, A P C, is less than half the bottom-width A m, proceed to find, as in Problems II. and III., the computed half-width M D′, and from thence the corrected half-width M D. But to find the corrected half-

width M C (which consists partly of an embankment), multiply the distance M D (just found) by the difference of the bottom-width and the estimated half-width, and divide the product by the estimated half-width, and the quotient is the corrected half-width M C.*

Ex. Let the bottom-width A B = 30 ft., the depth M m = 3 ft., the ratio of the slopes $1\frac{1}{2}$: 1, and the difference of level-reading 7 ft., at the distances of 25 and 24 ft. from the centre-stump on the surface-slope, and on the level respectively; required the corrected half-widths M D and M C.

By Prob. II. $3 \times 1\frac{1}{2} + \frac{30}{2} = 19\frac{1}{2}$ ft. = estimated half-width M D′.

By Prob. III. $\frac{19\frac{1}{2} \times 25}{24 - 7 \times 1\frac{1}{2}} = \frac{975}{27} = 36.11$ ft. = corrected half-width M D.

By Prob. IV. $\frac{36.11 \times (30 - 19\frac{1}{2})}{19\frac{1}{2}} = 19.44$ ft. = corrected half-width M C.

If the same symbols be used for the given parts in this example, as in Prob. III., and w = bottom-width: then

$$\text{M D} = \frac{b\,s}{l - r\,h} = \frac{19\frac{1}{2} \times 25}{24 - 7 \times 1\frac{1}{2}} = 36.11 \text{ ft.} = \text{corrected half-width.}$$

$$\text{M C} = \frac{(w - b)\,s}{l - r\,h} = \frac{(30 - 19\frac{1}{2})\,25}{24 - 7 \times 1\frac{1}{2}} = 19.44 \text{ ft.} = \text{corrected half-}$$

width.

NOTE. — This operation for finding M C, it will be seen, is different from the preceding one. See *Demonstration*.

Construction. The operation for this Problem is the same as that for Prob. III., excepting that A C is drawn parallel to B D.

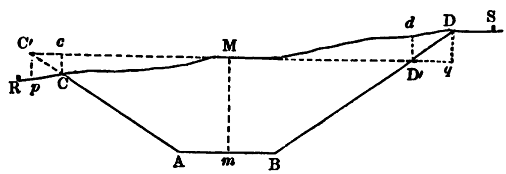
By reversing the cross-section, it will be readily seen that the same

* *Demonstration.* Draw A $c \perp$ to C′ D′ prolonged, and let pq, which is $\perp$ to p C′ D′, be the difference of level-readings at M and q: then, if a = Mm, Mc = Am = $\frac{1}{2}w$ = $\frac{1}{2}$ bottom-width, the other symbols being the same as in the Demonstration to Prob. III., $ar = c$C′ and C′ M = Mc − cC′ = $\frac{1}{2}w - ar$; whence, by the Demonstration above referred to, M C = $\frac{s(\frac{1}{2}w - ar)}{l - rh}$: but $b = \frac{1}{2}w + ar$, or $ar = b - \frac{1}{2}w$; this value being substituted in that of M C, gives $\frac{s(w - b)}{l - rh}$ = M C. Also $\frac{s(w - b)}{l - rh}$ = M C = $\frac{bs}{l - rh} \times \frac{w - b}{b}$, which is the rule given in words at length. *Q. E. D.*

calculation and construction will apply in the case of A P C, being a cutting, and P B D an embankment, observing that the corrected half-width for the cutting is the shorter distance, and *vice versâ.*

PROBLEM V.

To find the Width of the Cuttings when the Surface of the Ground is laterally very uneven.



Let A B D M C be a cross-section of the cuttings, M m the depth, C′ M = M D′ = estimated half-width, C D M the uneven surface of the ground, &c. The solution of this Problem will be best effected by giving an example in numbers.

Let A B = 30 ft., M m = 30 ft., and the ratio of the side-slopes as $1\frac{1}{2} : 1$; then M C′ = M D′ = $30 \times 1\frac{1}{2} + \frac{1}{2} \times 30 = 60$ ft. Measure from M horizontally the distance M d = 60 feet, the point d is directly above D′. Place levelling-staves at M and d, and observe the difference of the readings at M and d, which, in this case, is 7 ft.; whence $7 \times 1\frac{1}{2} = 10\frac{1}{2}$ ft. = approximate distance d D, and M d + d D = M D = $60 + 10\frac{1}{2} = 70\frac{1}{2}$ ft. Now place a levelling-staff D, and observe the reading, which is found to be 7.8 ft. greater than that at M, or 0.8 ft. greater than that at d; whence $0.8 \times 1\frac{1}{2} = 1.2$ ft., and consequently $70\frac{1}{2} + 1.2 = 71.7$ ft., which is a still nearer approximation to the true distance M D or M q, it being measured horizontally. The operation for finding M c, or M C measured horizontally, is the same as the preceding, excepting that the product of the stave-readings is subtracted from the estimated half-width, &c. In this manner the distance M c = horizontal distance M C is found to be 52.6 feet.

NOTE 1. — The widths R C, D S, of the side fences, must be added to the above results for the whole width.

NOTE 2. — If the difference of the stave-readings at M and d be very large, it will require three or four approximations similar to those given in the preceding example, to find the true corrected half-width.

Construction. Take the levels of the several undulations of the surface C M D, making C′ M D′ the *datum*-line, and draw the cross-section A B D M C by the methods already given.

By reversing the cross-section A B D M C, its application to an embankment is obvious, observing also to reverse the distances M C, M D, as previously noticed.

NOTE 3. — The truth of the method of approximation, used in this example, is too obvious to require a demonstration.

LEVEL-BOOK.

	No. of Stump.	Depth of Cuttings or Embankments.	Computed Half-width.	Corrected Half-widths for edge of Cutting or foot of Embankment.		Whole widths including Fences each Nine Feet in Width.
				North.	South.	
		Feet.	Feet.	Feet.	Feet.	Feet.
Cutting.	216	30.00	60.00	52.60	71.70	142.30
	217	3.00	19.50	19.44	36.11	73.55
	218	28.00	57.00	57.95	56.08	133.03
	219	19.68	44.42	44.42	44.42	106.82
	220	20.00	45.00	32.60	83.33	133.93
Embankt.	221	16.08	39.12	39.72	59.12	116.84
	222	30.00	60.00	54.24	68.05	140.29
	223	32.18	63.27	63.00	63.62	144.62

NOTE. — The depths of cuttings or embankments, in the 2d column of the preceding Level-Book, are found by calculation, or by carefully measuring them from the section by the vertical scale, but the latter method is not sufficiently correct. The computed half-widths in the 3d column, are found by Problems I. and II. The corrected half-widths, in columns 4th and 5th, by the five preceding Problems, according to the nature of the cuttings or embankments.

PROBLEM VI.

To find the Quantity of Land required for a projected Railway.

CASE I. — *In preparing the preliminary Estimates for a projected Railway, the Quantity of Land required for the Purpose is usually found without paying any Regard to the lateral Inclination of the Ground, by taking a considerable Length of the Section at once, especially if the Surface thereof have a regular Rise or Fall, and by measuring the Depth of the Ends of such Length with the vertical Scale.*

RULE. — Find the surface-widths, fences included, at each end of the given length, by Problems I. and II., add them together, multiply the sum by the length in chains, and divide the product by 1320 for the area in acres.

Ex. Let the length of the sectional surface be 18 chains, and the depths at the ends 22 and 38 feet, the bottom-width of the railway 33 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1 ; required the area of the surface, the width of the side-fences being 9 feet each.

By Prob. II. $w+2\,ar+2f=33+3\times22+2\times9=117$

$$
\begin{array}{r}
w+2\,br+2f=33+3\times38+2\times9=165\\
\overline{282}\\
18\\
1320\begin{cases}12|\overline{507.6}\\11|\overline{42.3}\end{cases}\\
\end{array}
$$

	A.	R.	P.
3.84545 =	3	3	15

or, by putting $l=$ given length, and taking half the sum of the widths, there will result

$$\frac{(w+r\,\overline{a+b}+2f)l}{660}=(33+1\tfrac{1}{2}\times\overline{22+38}+2\times9)\,\frac{18}{660}=3.84545 \text{ acres.}$$

CASE II. — *When the exact Quantity of Land for the Railway is required.*

RULE. — Take the whole widths, at the end of every chain, from the 6th column of the Level-Book, for the several widths; add continually together the first and last widths, and twice the sum of all the intermediate widths, and divide the whole sum by 1320 for the area in acres.

Ex. Required the area corresponding to the several widths in the Level-Book at the end of Prob. V.

$$
\begin{array}{r l}
73.55 & \\
133.03 & \\
106.82 & \\
133.93 & \\
116.84 & \\
\underline{140.29} & \\
704.46 & \\
\underline{2} & \\
1408.92 & = \text{twice sum of intermediate widths,}\\
142.30 & = \text{first width,}\\
144.62 & = \text{last width,}\\
1320\begin{cases}12|\overline{169.584}\\11|\overline{14.132}\end{cases} & \\
1.28473 & = 1\text{A. } 1\text{R. } 5\tfrac{1}{2}\text{P.} = \text{area required.}
\end{array}
$$

NOTE. — It is very common in practice to find the areas of the quantities of land, required from the several proprietors, by actual measurement from the 2 chain maps, made for the use of the contractors, after the several widths have been laid down thereon: copies being taken, at the same time, from the maps, on tracing paper, showing the position and quantity of land required from each proprietor.

SECTION III.

ON RAILWAY CUTTINGS IN GENERAL AND METHODS OF FINDING THEIR CONTENTS.

In preparing the preliminary estimates for a railway, the contents of the cuttings are usually found by tables for the purpose, the surface of the ground in the several cross-sections being assumed to be on a level with the centre of the line. But when power has been granted for constructing the line, the cross-sections are carefully taken at the end of every prominent variation of the surface of the ground, or, if consistent with accuracy, at the end of every one, two, or three chains in length; the several cross-sections are then plotted on a large scale (which may be done by the methods given in the preceding Problems), and their areas found by actual measurement; or reduced, where the surface of the ground is laterally sloping or curved, to horizontal sections, preparatory to finding the contents from the tables, by using the mean depths of the several sections, *which method is correct; but the mean depth, used in this method, cannot be accurately found in many cases without considerable calculation, which may be avoided by adopting the methods given in* Prob. II. p. 46. Some take the mean of every two succeeding sections, and others use a mean of the mean depths as the basis for a mean area, *both of which methods are very inaccurate; especially where the areas of the extreme sections differ greatly.* The magnitude of the errors in both cases, will be pointed out in the investigations at the end of these Problems.

On existing Earthwork Tables.

Tables for this purpose have been published by Sir John M'Neill, Mr. Bidder, Mr. Bashforth, Messrs. Sibley and Rutherford, and others; all of which are well adapted for finding the contents of cuttings, assuming the surface of the ground to be laterally level with respect to the direction of the cutting. But none of these tables are properly adapted to the finding of the contents from sectional areas, *i. e.* from the areas of working drawings, excepting Mr. Bashforth's tables; *but his mathematical investigation of the rule for using them in finding the contents from working drawings, where the surface of the ground is laterally sloping or curved, is founded on a false assumption, and, therefore, his results are erroneous, and especially so where the sectional areas differ considerably.* (See Preface, and the Investigations at the end of these Problems.)

THE GENERAL EARTHWORK TABLE.

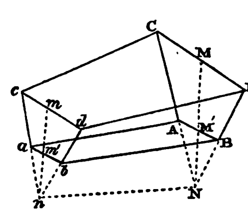
(*At the end of the book.*)

This Table, with the help of the Auxiliary Earthwork Tables, Nos. 1 and 2, on the same sheet, possesses the advantage of being general for all varieties of slopes and bottom-widths in common use, as well as for decimal parts of feet in the depths. It may also, with a very trifling preliminary calculation, be made to extend to every variety of bottom-width and ratio of slope that can occur, if even the slopes of the two sides differ in the same cutting; and with the help of a table of square roots it will apply, with all attainable mathematical accuracy, to cuttings where the surface of the ground is uneven. The investigation of the method of forming the Tables and using them, will be given at the end of these Problems. The contents in the general Table, and those in Table No. 2., are calculated to the nearest unit for one chain in length, and checked by differences; the side-slopes being assumed to be extended till they intersect. The auxiliary Table, No. 1., gives the depths of the intersection of the side-slopes below the balance-line, and the corresponding number of cubic yards to be deducted from the contents for each chain in length.

PROBLEM I.

CASE I.—*To find the Contents of Cuttings by the general Earthwork Table, and the Auxiliary Table,* No. I., *at the end of the Book.*

Let A B b d c C be a cutting, A B = a b = bottom-width on the formation level, M M′ and m m' the perpendicular depths at the middle of the two ends of the cutting; A C, B D, a c, b d the side-slopes, which, being prolonged two and two, will meet at the points N and n; also M M′ and m m', being prolonged, will meet at the same points. The distance M′ N = m' n in feet, and decimals is given in the Auxiliary Earthwork Table, No. 1., for all bottom-widths



and ratios of slopes in common use, at which distance a line must be ruled on the section, parallel to the balance-line, or at the same distance + 2 feet from the line of the rails, in which latter case the balance-line need not be drawn. From the line thus ruled, the depths of the cutting must be measured to adapt them to the General Earthwork Table; or a mark might be made on the vertical scale with Indian ink (which is easily washed off) at the same distance, which mark might then be applied to the line of the rails in measuring off the

depths. For measuring the depths of embankments, the line must be ruled at the same distance — 2 feet above the line of the rails. When the several quantities of a cutting or embankment have been taken from the Table, and their sum multiplied by the ratio of the slopes, the cubic yards to be deducted for each chain in length, for the particular bottom width and ratio of slopes, must be taken from Table No. 1. and multiplied by the whole length of the cutting, and the product, being subtracted from the result obtained from the General Table, will give the content of the cutting in cubic yards.

The method of using the Tables will best appear from the following examples: —

Ex. 1. Let the several depths of a railway cutting to the intersection of the slopes, at the end of every chain, be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting in cubic yards.

NOTE. — By the Table No. 1. the depth to be added to the depths of the cutting, for bottom width 30 ft. and ratio of slopes $1\frac{1}{2}$ to 1, is 10 ft., therefore the line from which the depths in the annexed table are measured is $10+2=12$ ft. below the line of the rails. The corresponding number of cubic yards to be subtracted is carried to two places of decimals, or, if the nearest whole number had been taken, the quantity would have been in excess or defect by several cubic yards, when the cutting is of a considerable length.

Dist. in Chains.	Depths.	Qts. per Table.
0	10	
1.00	29	1003
2.00	32	2276
3.00	33	2582
4.00	39	3175
5.00	35	3350
6.00	10	1365
For slope 1 to 1..		13751
—— $\frac{1}{2}$ to 1..		6875.5
—— $1\frac{1}{2}$ to 1..		20626.5
Subtract 366.67 × 6	=	2200
Content in cubic yds.	=	18426.5

Ex. 2. Let the several depths to the intersection of the slopes and their distances be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting.

NOTE. — When any of the distances is greater or less than 1 chain, the corresponding quantity must be multiplied by that distance; as in the cases of the distances between the depths 45 and 50, and between 30 and 10, the former distance being 3 chains and the latter 60 links.

Dist. in Chains.	Depths.	Dist. greater than 1 Chain.	Qts. per Table.
0	10		
1.00	20		570
2.00	16		795
3.00	25		1044
4.00	32		1996
5.00	39		3091
6.00	45		4319
9.00	50	3 × 5520	16560
10.00	40		4971
11.00	30		3015
11.60	10	.60 × 1059	635.4
For slopes 1 to 1			36996.4
—— $\frac{1}{2}$ to 1			18498.2
—— $1\frac{1}{2}$ to 1			55494.6
366.67 × 11.60 =			4253.4
Content in cubic yards.....			51241.2

Ex. 3. The depth of a cutting to the intersection of the slopes, and their distances in feet are as in the annexed table, the bottom width is 36 feet, and the ratio of the slopes 2 to 1; required the content of the cutting in cubic yards.

Dist. in Feet.	Depths.	Qts. × by Length.	Products.
0	39		
100	61	6210 × 100	621000
188	50	7554 × 88	664752
178	37	4660 × 90	419400
For slopes 1 to 1...........			1705152
			2
—— 2 to 1......			3410304
		396 × 278 =	110088
			66)3300216
Content in cubic yards			50003

NOTE. — When the distances are given in feet, the quantities from the General Table must be multiplied by their respective distances; also the quantity from Table No. 1. must be multiplied by the whole distance, and the final result divided by 66, as in the annexed example. See Demonstration at the end of these Problems.

CASE II.

To find the Contents of Cuttings by the Tables, when the Depths are given in Feet and Decimals of Feet.

RULE. Let any two succeeding depths be denoted by a and b, and let the decimal parts of the depths be respectively denoted by a' and b'; find the quantity corresponding to a and b from the General Table, as in the former case; then,

$\frac{2a+b}{10}$, or its nearest whole number, and the decimal a' will show the number of cubic yards to be added in Table No. 2., and

$\frac{2b+a}{10}$, or its nearest whole number, and the decimal b' will show the cubic yards to be added in the same Table.

Ex. 1. Let the depths to the intersection of the slopes be 61.6, and 39.4 feet, their distance 1 chain, the bottom width 36 feet, and the ratio of slopes 2 to 1; required the content of the cutting in cubic yards.

$$\text{Put } 61 = a, \text{ its decimal } .6 = a'$$
$$39 = b, \text{ its decimal } .4 = b'.$$

Then the depths a and b per General Table give -	6210
$\frac{2a+b}{10}=16.1$, or its nearest whole No. 16 and (a') .6 per Table No. 2. - - - - - -	78
$\frac{2b+a}{10}=13.9$, or its nearest whole No. 14 and (b) .4 per Table No. 2. - - - - - -	46
For slopes 1 to 1 - -	6334
	2
For slopes 2 to 1 - -	12668
By Table No. 1., for bottom width 36 feet, and ratio of slopes 2 to 1, No. of cubic yds. to be deducted - -	396
Content in cubic yards	12272

NOTE 1. — Care must be taken to use the decimal a' with $\frac{2a+b}{10}$, and the decimal b' with $\frac{2b+a}{10}$, in finding the quantities in Table No. 2., as a mistake might easily be made in this matter, which would lead to an erroneous result.

2. If any two succeeding depths be nearly equal, and the sum of their decimal parts be together equal to 1 foot, or nearly so, by adding 1 foot to the lesser depth, and rejecting the decimal in the larger depth, and using the depths thus altered as whole numbers, a result sufficiently correct will be obtained; as in the following example.

Ex. 2. Let the depths of a cutting be 50.29 and 48.7 feet, their distance 1 chain, the bottom width 33 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content.

By adding the decimal .29, in the larger depth, to 48.7, the depths may be called 50 and 49, for which, by the General Table, the content is - - - - 5990 for 1 to 1

	5990 for 1 to 1
	2995 for $\frac{1}{2}$ to 1
	8985 for $1\frac{1}{2}$ to 1
From which deduct, from Table No. 1.	443.67
	8541.33 cubic yards.

NOTE 1. — The results of all the examples in the two preceding cases only differ by a very small fraction of a cubic yard from the true contents obtained by calculation from formula (1.), page 51. The method of finding the contents to decimals, or tenths of a foot in the depths having been particularly discussed, on account of its utility in finding the contents for actual contract-work from the working drawings; in which, as great accuracy is required, the contents should be found to two places of decimals, or to $\frac{1}{100}$ths of a foot in the depths, as in Case II. of the following Problem.

2. When one or both of the given depths exceed the limits of the table, find the content corresponding to half the two depths, and four times the result will be the content required.

PROBLEM II.

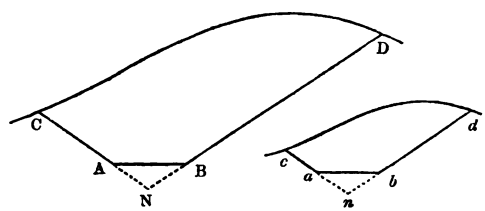
CASE I.

To find the Content of a Cutting between two Cross Sections the Areas of which, to the Intersection of the Slopes, the Length, the Bottom Width, and the Ratio of the Slopes, being given.

RULE. Find the square roots of the given areas either by a table of square roots*, or by actual extraction; with these roots, as depths, proceed to find the content from the General Table, as in Prob. I., from which deduct the quantity corresponding to the given bottom width and ratio of slopes from Table No. 1., and multiply the remainder by the length for the content.

NOTE. — If the length be given in feet, multiply the content found by the above rule by the feet, and divide by 66 for the content.

Ex. 1. Let the areas of the two ends of a cutting be 5141 and 1444 square feet, the bottom width 30 feet, the length 1.60 chains, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting in cubic yards.



Let the annexed figures be the two cross-sections, the side slopes C A, D B; *c a*, *d b*, being prolonged till they meet at N and *n*, the area C N D = 5141, the area *c n d* = 1444; and since the bottom-width A B = *a b*, and the ratio of the slopes are given, the solidity of the prism, the ends of which are A B N = *a b n*, is given in Table No. 1., and is to be deducted from the content found by the General Table, as in Prob. I.

The square roots of 5141 and 1444 are -	71.7 and 38
By General Table, for 71 and 38 - -	7483
By Table No. 2., for $\frac{71 \times 2 + 38}{10} =$ 18 and .7 -	103
Content to intersection of slopes - -	7586
By Table No. 1., for bottom width 30, and slopes $1\frac{1}{2}$ to 1	366.67
Content for 1 chain in length - -	7220.33
	1.60
Content for 1.60 chains in length - -	11552.528 cubic yards.

* Barlow's Tables are the best for this purpose.

CASE II. When great accuracy is required, especially in the measurement of contract work, the second decimals, or $\frac{1}{100}$ths, of feet must be considered in the calculation by taking for them $\frac{1}{10}$th of their corresponding quantities in Table No. 2.

Ex. Let the areas of the several cross sections to the intersection of the slopes, and their distances, be as in the annexed table, the bottom width 36 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting in cubic yards.

Dist. in Chains.	Areas in Sq. Ft.	$\sqrt{\text{Areas}}$.	Products.	Contents.
0	2161	46.59		
1.00	3759	61.31	...	7147
2.00	5141	71.7	...	10832
4.20	4100	64.03	11271 × 2.20	24796
6.00	4221	64.97	10170 × 1.80	18306
8.00	3136	56.	8959 × 2.00	17918
9.00	2727	52.22	...	7160
				86159
From Table No. 1. for bottom width 36 ft. and slopes $1\frac{1}{2}$ to 1, 528 × 9.00....				4752
Content in cubic yards....................				81407

PROBLEM III.

To find the Content of a Railway Cutting when the Slopes of the two Sides are different.

CASE I. *When the Depths are given.*

RULE. — Find the content by the Tables, as in the two preceding Problems, multiply the result by half the sum of the side slopes, and from the product deduct half the sum of the cubic yards, corresponding to the given bottom width and the ratios of the slopes of the two sides, multiplied by the whole length of the cutting, and the remainder is the content in cubic yards.

Ex. Let the depths and their distances be as in the annexed table, the bottom width 36 feet, and the ratios of the slopes of the two sides 1 to 1, and $1\frac{1}{2}$ to 1; required the content of the cutting.

Dist. in Chains.	Depths.	Qts. per Tables.
0	61	
1.00	52	7820
2.00	44.46	5473
For slopes 1 to 1......13293		
—— $\frac{1}{4}$ to 1...... 3323.25		
16616.25		
$\frac{1}{2}$(528 + 792) × 2 1320		
Content................15296.25		

NOTE. — Here the half sum of the first terms of the ratios of the slopes, viz. $\frac{1}{2}(1 + 1\frac{1}{2})$ $= 1\frac{1}{4}$ is the number by which the sum of the quantities, per Tables, is to be multiplied, or $\frac{1}{4}$ added, as in the operation: and half the sum of the cubic yards, for the given bottom width and ratios of the two slopes, is multiplied by the whole length of the cutting, and deducted, agreeable to the rule.

CASE II. *When the Sectional Areas are given.*

RULE. — Find the content from the Tables as in Prob. II., and deduct for the quantity below the cutting, as in Case I. of this Problem.

NOTE. — This Rule seems too obvious to require an example.

PROBLEM IV.

To apply the General Table and Table No. 2., to such Bottom Widths and Ratios of Slopes as are not found in Table No. 1.

Put $w =$ bottom width, and r to 1, the ratio of slopes.

Then $\frac{w}{2r} =$ feet to be added to depth of cutting, or distance from bottom of cutting to the intersection of slopes.

And $\frac{11}{18}\frac{w^2}{r} =$ cubic yards to be deducted for each chain in length.

Ex. If the bottom width be 28 feet, and the ratio of slopes $1\frac{1}{4}$ to 1; then $w \div 2r = 28 \div 2 \times 1\frac{1}{4} = 11\frac{1}{5} = 11.2$ feet $=$ distance from bottom of cutting to intersection of slopes, and $11w^2 \div 18r = 11 \times 28^2 \div 18 \times 1\frac{1}{4} = 383.29$ cubic yards to be deducted for each chain in length.

PROBLEM V.

To find the Content of the Cutting of a Tunnel.

1. *When the length is given in yards,* and the width and height in feet.

RULE. — Multiply continually together the length, width, and mean height, and divide the product by 9.

2. *When the length is given in chains, and the width and height in feet.*

RULE. — Multiply the continued product of the length, width and mean height by 22, and divide by 9.

Ex. The length of cutting of a tunnel is 1053 yards, its width 30 feet, and mean height 32 feet; required the content.

$$\frac{1053 \times 30 \times 32}{9} = 11232 \text{ cubic yards.}$$

The following Example shows the Magnitude of the Errors of several Methods, practically used, for calculating the Contents of Cuttings.

Ex. The areas of the two cross sections of a cutting, to the intersection of the slopes, are 10324 and 400 square feet, their

distance 4 chains, the bottom width 36 feet, and the ratio of the slopes 1 to 1; required the content of the cutting by the General Table, &c., and by the erroneous methods practically used.

NOTE. — The great difference of the sectional areas in this example, shows very prominently the errors of several methods of finding the contents of cuttings: at the same time, it is proper to remark, that similar differences in the sectional areas very frequently occur in practice, and that the erroneous methods give the contents very near the truth *only* when these areas are nearly equal.

$\left.\begin{array}{l}\sqrt{10324} = 101.607 \\ \sqrt{\ \ 400} = \ \ 20\end{array}\right\}$ Content by General Tab., &c. 10394

Deduction from Tab. No. 1.	792
Content for 1 chain in length	9602
	4
True content for 4 chains in length	38408 cub. yds.

By Mr. Bashforth's Method.

(1.) By Table No. 1., the depth below the formation-level to the meeting of the slopes is 18 ft.; hence the area of the triangle below it is $\frac{1}{2}(36 \times 18) = 324$ sq. ft., which, deducted from the given sectional areas, gives 10000 and 76 sq. ft. for the sectional areas, as used by Mr. Bashforth.

Whence $\left.\begin{array}{l}\sqrt{10000} = 100 \\ \sqrt{\ \ \ 76} = \ \ 8.718\end{array}\right\}$ Content by Gen. Tab., &c. 8920.4

	4
Content for 4 chains in length	35681.6 c. y.
Error in defect, compared with the true content given above, *being above* $7\frac{1}{2}$ *per cent., by Mr. B.'s method.*	2726.4

(2.) *By taking a mean of the areas for a mean Section.*

$\frac{1}{2}(10000 + 76) \times \frac{22}{9} \times 4 = 49260.4$ cubic yards; *which exceeds the true content by* 10852.4 *cubic yards, being above* 22 *per cent. in excess.*

(3.) *By taking a mean depth the error in defect is just half of the preceding error, or nearly* 11 *per cent.*

OBSERVATIONS IMPORTANT TO THOSE CONCERNED IN THE CONSTRUCTION OF RAILWAYS, ON THE ERRONEOUS METHODS OF CALCULATING THE CONTENTS OF CUTTINGS, WHERE THE SURFACE OF THE GROUND IS UNEVEN; ALSO ON THE METHOD GIVEN IN THIS WORK.

THE magnitude of the errors of the methods (1.), (2.) and (3.), in the last example, for calculating the contents of cuttings, where the surface of the ground is uneven, from sectional areas, is strikingly apparent; and since the Tables of Sir John M'Neill, Mr. Bidder, and others, are not accompanied by any directions for their practical application, in this particular case, one or other of the last two of these defective methods is still used by the great majority of engineers and contractors: thus causing continual disputes concerning the contents, in consequence of the different parties using irreconcileable methods, some preferring one and some the other, as giving, in their respective judgments, the *true* content.

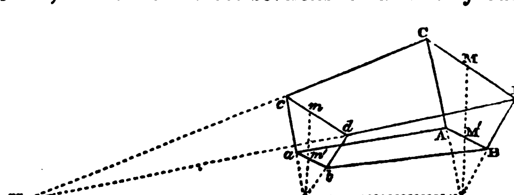
Mr. Bashforth's method of finding the contents from sectional areas (lately published) is also unfortunately erroneous; as has been shown in (1.) last example, although his method has the *appearance of mathematical demonstration for its basis.*

It is therefore the interest of those concerned in the construction of railways to adopt the method given in this work for finding the contents of cuttings, where the surface of the ground is laterally sloping, &c., as no other method combining all attainable mathematical accuracy has, to my knowledge, been yet published: those already published referring only to cuttings, where the surface of the ground is either level, or the sections thereof assumed to be reduced to a level, which, in many cases, is a work of great labour to perform accurately. See the following Investigations.

INVESTIGATION

OF THE CONSTRUCTION AND USE OF THE GENERAL AND AUXILIARY TABLES, AND OF THE ERRORS OF THE METHODS OF MR. BASHFORTH AND OTHERS.

Let A B D C, a b d c, be vertical cross sections of a railway cutting, the surface widths C D, c d being horizontal, A C c a, B D d b, the side-slopes, A B $= a$ b the bottom width. Prolong the planes



of the side slopes downwards and to the left, till they meet in N n V, and also meet the prolongation of the plane C D d c in c V, d V. Because A B = a b, the cross sections are $\perp$ to V N. Let fall the $\perp$s N M, n m on C D, c d respectively, bisecting them in M and m, and also bisecting A B, a b in M$'$ and m'. Put M N = a, m n = b, N n = A a = B b = l, A B = a b = w, all in feet, and the ratio of the slopes, *i. e.*, C M : M N :: r : 1. Then Δ C D N = $a^2 r$, Δ c d n = $b^2 r$. By similar figures $a : b ::$ N V : N V $- l$, whence N V = $\frac{a\,l}{a-b}$,

$$\text{Pyramid V C N D} = \tfrac{1}{3}\ \text{N V} \times a^2 r,$$

$$\ldots\ldots\ \text{V}\ c\ d\ n = \tfrac{1}{3}(\text{N V} - l)\, b^2 r;\ \text{whence}$$

Frustum C D N n c d = $\frac{1}{3}$ N V $\times$ $a^2 r - \frac{1}{3}$ (N V $- l$) $b^2 r$ = $\frac{1}{3} r\, (\overline{a^2 - b^2}\ \text{N V} + b^2 l)$ = (by substituting the value of N V)

$$\tfrac{1}{3}\, r\, l\, (a^2 + a\,b + b^2)\ \text{cubic feet.}$$

$$= \frac{r\,l}{81}\,(a^2 + a\,b + b^2)\ \text{cubic yards} \quad - \quad - \quad - \quad (1.)$$

In the General Table $r = 1$, and $l = 1$ chain = 66 feet; and $\therefore$ the solidity

$$S = \frac{22}{27}\,(a^2 + a\,b + b^2)\ \text{cubic yards} \quad - \quad - \quad (2.)$$

wherein a and b have all integral values from 0 to 72.

Auxiliary Table, No. 1.

$$r : 1 :: \tfrac{1}{2} w\ (= \text{A M}') : \text{N M}' = \frac{w}{2\,r}, \quad - \quad - \quad - \quad (3.)$$

$$\text{whence}\ \Delta\ \text{A B N} = \Delta\, a\, b\, n = \tfrac{1}{2}\,\text{A B} \times \text{N M}' = \frac{w^2}{4\,r}, \quad - \quad - \quad - \quad (4.)$$

$$\therefore\ \text{Prism A B N}\ n\ a\ b = \frac{w^2\, l}{4\ r}\ \text{sq. ft.} = \frac{w^2\, l}{108\ r} = \text{cubic yds.} \quad (5.)$$

From (3.) and (5.) the depths to be added, and the cubic yards to be deducted in Table No. 1., are calculated, by taking $l = 66$ feet, and w and r all the values most commonly used in practice.

$$\therefore\ \frac{w^2\, l}{108\ r} = \frac{11\ w^2}{18\ r} = \text{cubic yards to be deducted, Table No. 1.} \quad (6.)$$

$$\therefore\ \frac{11\ w^2\,\text{L}}{18\ r} = \text{cubic yards to be deducted for the length L.}$$

Let $\Sigma =$ sum of all the solidities S, S′, &c. (per General Table), the length of which is L, and $\kappa = \frac{H\,w^2}{18\,r}$ from (6.), then $\Sigma\,r =$ sum of solidities for slopes r to 1, from which deduct $\kappa \times L$, and there results

$\Sigma\,r - \kappa\,L =$ cubic yards in the whole cutting - - (7.)

Whence the method of finding the contents of cutting in Prob. I.

COR. 1. The content of a cutting having only two given depths is $(S\,r - \kappa)\,L$ cubic yds. - - - - - (8.)

COR. 2. If the ratio of the slopes of the two sides of a cutting be, r to 1 and ρ to 1, and k, κ the corresponding cubic yards to be deducted. (Table No. 1.)

$$\tfrac{1}{2}\{\Sigma\,(r+\rho) - L\,(k+\kappa)\} \text{ cubic yards.}$$

Let in (2.) a and b, hitherto supposed to be integral numbers, have the increments or decimals α and β, so that a and b become respectively $a+\alpha$ and $b+\beta$, which being substituted for a and b, neglecting the squares and product of α and β, as being comparatively small, there results:

$$\frac{22}{27}(a^2 + a\,b + b^2) + \frac{22}{27}(2\,a + b\,)\alpha + \frac{22}{27}(2b + a)\,\beta \quad - \quad (9.)$$

By subtracting (2.) from (9.), there results the sum of

$$\frac{22}{27}(2\,a + b)\,\alpha \text{ and } \frac{22}{27}\,(2\,b + a\,)\,\beta, \text{ or}$$

$$\frac{22}{27}\left(\frac{2\,a + b}{10}\right) 10\,\alpha \text{ and } \frac{22}{27}\left(\frac{2b + a}{10}\right) 10\,\beta \quad - \quad (10.)$$

which are the increments of the formula (2.) arising from the addition of the decimal parts α and β to the depths a and b, from which Table No. 2. has been calculated, the quantities $2\,a+b$, and $2\,b+a$ being divided by 10, to prevent a too great extension of the Table; and the corresponding factors α and β multiplied by the same number, that their product might retain their original value, the decimal points being still affixed to the values of α and β in the horizontal line at the top of the Table.

In consulting Table No. 2. the nearest whole numbers to $\frac{2\,a+b}{10}$ and $\frac{2\,b+a}{10}$ are taken, as the small errors, thus resulting, will usually balance one another in a long cutting, and can never in any case amount to much.

Let the plane V C D revolve on the side V C, so that the lines C D, $c\,d$, may incline from their hitherto assumed horizontal posi-

tion, thus making the side slopes unequal; and let the area C D N = A, and the area $c\ d\ n$ = B, the other symbols being the same as before, then

$$\sqrt{A} : \sqrt{B} :: VN : VN - l, \text{ whence } VN = \frac{l\sqrt{A}}{\sqrt{A} - \sqrt{B}}$$

Pyramid V C D N $= \frac{1}{3}$ V N $\times$ A, Pyramid V $c\ d\ n = \frac{1}{3}$ (V N $- l$) $\times$ B, whence the frustum C D N $n\ c\ d$ = S = $\frac{1}{3}$ V N $\times$ A $-$ $\frac{1}{3}$ (V N $- l$) $\times$ B $= \frac{1}{3}\ (\overline{A - B} \times VN + B\,l)$

$$\text{or } S = \tfrac{1}{81}\,l\,(A + B + \sqrt{A \times B}), \text{ in cubic yards, and taking } l = 66\text{ft.}$$

$$= \frac{22}{27}\,(A + B + \sqrt{A} \times \sqrt{B}) \text{ cubic yards} \quad - \quad - \quad (11.)$$

Therefore the Table must be consulted for depths $\sqrt{A}$ and $\sqrt{B}$, the ratio of the slopes being included; for if a and b be the mean depths for the areas A and B respectively, the slopes being r to 1, then

$a^2 r = A$, $b^2 r = B$ and $\frac{22}{27}\,(A + B + \sqrt{A} \times \sqrt{B}) = \frac{22\,r}{27}\,(a^2 + a\,b + b^2)$.

Therefore in reality the Table is consulted for the depths $a\sqrt{r} = \sqrt{A}$, and $b\sqrt{r} = \sqrt{B}$.

If Σ' = sum of all the solidities S, S′, &c. for depths $\sqrt{A}$ and $\sqrt{B}$, &c., and length L; then by subtracting κ L, as in (7.) there results

$\Sigma' - \kappa$ L cubic yards in the whole cutting - - - - - - - (12.)

Whence the rule in Problem II.

COR. 1. The content of a cutting, having only two given sectional areas, is (S $- \kappa$) L cubic yards - - - - - - - - - - (13.)

COR. 2. Formula (11.), (12.), and (13.), will evidently hold, if, instead of the straight lines C D, $c\ d$, being the surface edges of the cross sections C D N, $c\ d\ n$, of the cutting, these lines be the chords of similar curves forming the surface edges of the cross sections, and the similarly situated points in the two curves be joined by right lines (which prolonged will all meet in the vertex V), thus forming the surface of the cutting, a necessary condition in taking cross sections; or the small inequalities of the earth's surface between the cross sections must be balanced, as nearly as can be judged by the eye, so that the surface may fulfil this condition, in order that all attainable mathematical accuracy may be arrived at in finding the contents.

THE ERRORS OF THE METHODS OF FINDING THE CONTENTS OF RAILWAY CUTTINGS BY MR. BASHFORTH AND OTHERS.

Mr. Bashforth, in his investigations for finding the contents of railway cuttings, where the surface of the ground is level, or assumed to be so, includes the Prism A B N *n b a*, and afterwards deducts it, which is mathematically accurate. But in finding the contents from sectional areas, where the surface of the ground is laterally sloping, or uneven, he altogether leaves out the above-named prism; for he says (Art. 15. p. 11. of his work), "The only way to proceed in such a case is,—find the areas of the cross sections in square feet, take out of Barlow's Tables the square root of each, and treat these square roots precisely as if they had been measured heights, excepting that there will be nothing to be deducted for the prism, and no multiplication for the slopes, these having been already accounted for in finding the areas."

Let A′ and B′ be the sectional areas C D B A, *c d b a* respectively, as used by Mr. B., and l = N *n* = length of the cutting, (fig. page 50.)*, then, according to his method, the content of the cutting is

$$\frac{l}{81}(A' + \sqrt{A' \times B'} + B') \text{ cubic yards } - - - - - (14.)$$

which is erroneous in every case, except where the areas represented by A′ and B′ are similar and equal. For when the equal areas A B N, *a b n* are omitted, the areas represented by A′ and B′ are dissimilar, and ∴ the proportion $\sqrt{A'} : \sqrt{B'} :: NV : nV$, on which Mr. B's rule is founded, does not hold, it being true only in respect to the areas C D N, *c d n*, because the solids V C D N, V *c d n* are similar. Moreover, if the plane V C D revolve on the side C D so as to bring the lines *c d*, *a b*, to coincidence, or almost to coincidence, in the points *b* and *d*, the dissimilarity of the planes C D B A, *c d b a* will become more strikingly evident, whereas with the addition of the Δs A B N, *a b n*, they still remain similar.

Let Δ A B N = *a b n* = α, then $A' + \alpha$ and $B' + \alpha$ are the sectional areas corresponding to A and B in formula (11.) of my investigation, whence by that formula the true content, including the prism A B N *n b a*, is

$$\frac{l}{81}(A' + B' + 2\,\alpha + \sqrt{\overline{A' + \alpha} \times \overline{B' + \alpha}}) \text{ cubic yards.}$$

But the solidity of the prism is $\dfrac{\alpha\, l}{27} = \dfrac{3\,\alpha\, l}{81}$ cubic yards.

* The lines C D, *c d* are here assumed to be inclined to the horizon, and to be either right lines or similar curves.

Whence the content of the prismoid or cutting is

$$\frac{l}{81}(A' + B' + 2a + \sqrt{\overline{A' + a} \times \overline{B' + a}} - 3a), \text{ or}$$

$$\frac{l}{81}(A' + B' + \sqrt{\overline{A' + a} \times \overline{B' + a}} - a),$$

from which subtract the content according to Mr. B. (14.), and there results

$$\frac{l}{81}(\sqrt{\overline{A' + a} \times \overline{B' + a}} - \sqrt{A' \times B'} - a) = \text{error in defect.}$$

Let $A' = \frac{1}{4}a$, and $B' = 36a$, which is a case that may occur in practice, then

$$\frac{l}{81}(\sqrt{\overline{A' + a} \times \overline{B' + a}} - \sqrt{A' \times B'} - a) = \frac{a\,l}{81}(\tfrac{1}{2}\sqrt{185} - 3 - 1)$$

$$= \frac{a\,l}{81} \times 2.8007 = \text{error in defect by Mr. B.'s method.}$$

and $\frac{l}{81}(A' + \sqrt{A' \times B'} + B') = \frac{a\,l}{81}(\frac{1}{4} + 3 + 36) = \frac{a\,l}{81} \times 39\frac{1}{4} =$ whole content by Mr. B.'s method.

$\therefore$ whole content : error in defect :: $\frac{a\,l}{81} \times 39\frac{1}{4} : \frac{a\,l}{81} \times 2.8007.$

$$39\tfrac{1}{4} : 2.8007.$$

$$100 : 7\tfrac{1}{9}, \text{ or } 7\tfrac{1}{9} \textit{ per cent,}$$

the error in defect - - - - - - - - - (15.)

See *Ex.* at the end of Prob. X., where the error of Mr. Bashforth's method is shown by giving actual areas.

NOTE 1. — The errors of this method are not so prominent where the sectional areas approach near to equality, as in the case of the Burnley cross sections, in the *Ex.* Art. 20. page 13. of Mr. B.'s work : his method, however, is erroneous to a greater or lesser extent, in every case, except where the sectional areas are equal and similar ; for they cannot be similar without being equal, while the bottom width remains the same.

Mr. Bashforth says, in defence of his method of finding the contents of cuttings from sectional areas, in the *Mechanic's Magazine* for Sept. 11. 1847, p. 249, "In the case of contract estimates, numerous cross sections ought to be taken ; and it matters little whether we start from the intersection of the slopes or the formation level." *The above-noticed error* (15.) *shows that "it matters" so much as* $7\frac{1}{9}$ *per cent. in defect.* Besides, there is no need of taking "numerous

cross sections," as Mr. B. recommends, especially where the surface of the ground is laterally sloping like a geometrical plane, or curved like a conical surface, one or other of which cases very frequently occur, so that cross sections, taken at a considerable distance from one another to the intersection of the slopes, may be considered as similar, or so very nearly so, as not to induce any important mathematical error, which conditions may be easily determined by the eye. Moreover, the expense and trouble of taking "numerous cross sections," plotting them, and finding their areas, are very considerable; and, therefore, ought to be avoided, together with all erroneous methods, such as Mr. B.'s, of finding the contents of cuttings, as his "numerous cross sections" only tend to diminish the errors, without wholly getting rid of them. (See p. 42. and Cor. 2. p. 53.)

The Error of the Method of finding Contents by mean Areas.

Let A and B be the areas of two cross sections of a cutting to the intersection of the slopes, and l its length: then the mean area is $\frac{1}{2}(A + B)$, and the content in cubic yards is $\frac{1}{27} l \times \frac{1}{2}(A + B) = \frac{1}{81} l \times \frac{3}{2}(A + B)$; from which subtract the true content, equation (11.), p. 53., and there results

$\frac{1}{81} l \left\{\frac{3}{2}(A + B) - (A + B + \sqrt{AB})\right\} = \frac{1}{162} l (\sqrt{A} - \sqrt{B}) =$ *error in excess.*

Which error is very great when the areas A and B differ considerably. See (2.) p. 49.

The Error of the Method of finding the Content by mean Depths.

Let a and b be the depths of two cross sections to the intersection of the slopes, the ratio of which is r to 1, and $l =$ length of the cutting; then $\frac{1}{2}(a + b) =$ mean depth, $\frac{1}{4} r (a + b)^2 =$ mean area, and the content in cubic yards $= \frac{1}{27} l \times \frac{1}{4} r (a + b)^2 = \frac{1}{81} r l \times \frac{3}{4} (a + b)^2$;

which subtract from equation (1.), p. 51, and there results

$\frac{1}{81} r l \left\{a^2 + ab + b^2 - \frac{3}{4}(a + b)^2\right\} = \frac{1}{324} r l (a - b)^2 =$ error in defect:

Which error is very considerable, when the depths a and b differ greatly.

NOTE. The errors of these methods, as here shown, are the same as if the areas and depths had only extended to the formation level, since a common quantity, *i. e.* the prism below the formation level, is here included, and afterwards excluded by taking the differences.

SECTION IV.

TUNNELLING.

(1.) PREVIOUS to setting out the earthwork of a tunnel, the levelling operation must be repeated with great care, which should also be checked by the method I have given in Art. (22.) Section I. Part X. Nesbit's Surveying, especially if the tunnel pass under a very high summit: for, if the section be incorrect, the gradient or gradients, on which the tunnel is formed, will not meet at the points shown thereon, and thus embarrass the mining operation.

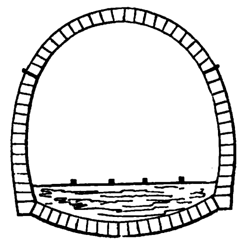
(2.) If the tunnel is formed upon a single gradient, the gradient must incline to one of the extremities of the tunnel, in order to discharge the water generated therein. Strong poles or masts must be firmly fixed on the surface, in the intended direction of the tunnel, of which one must be on the summit of the hill; at which place a temporary observatory is frequently erected, especially if the summit be a very high one, and the tunnel a very long one; but such an observatory can be of no use, except as a shed to shelter the engineers occasionally when they are superintending the mining operations in stormy weather. Shafts must be sunk at the distance of four or five chains from one another, in the direction of the poles (and observatory, if there be one), in order to ventilate the tunnel, as well as to check the accuracy of the work, as it proceeds. If the tunnel be a long one, it would be preferable, if convenient, to form it on two gradients, inclining to its opposite extremities to liberate the water, and thus to aid the mining operation, which is commonly commenced at both ends of the tunnel at the same time.

(3.) When it is necessary to have a curve in the direction of a part or of the whole of the tunnel, that direction must be carefully laid down on the surface, by the methods given in Section I., making allowance for acclivities and declivities, poles or masts being fixed therein, as pointed out in Art. (2.), that the shafts may be sunk so as to meet the mining operations of the tunnel, as well as to check their accuracy in point of direction, and this will be the more especially necessary in the curved part of the tunnel.

(4.) The mining operation of the tunnel should commence when the depth of the cuttings at each end is 60 feet. The width and depth of the excavation of a tunnel, on the narrow gauge, should be about 30 feet each, and must be dug 5 or 6 feet below the intended line of the rails, to give space for the inverted arch and the ballasting, excepting where the excavation is made through rock sufficiently

hard to form the side-walls of the tunnel, in which case 22 or 24 feet in width, and about 26 feet in height, will be sufficient, the excavation in this case being terminated below by the balance line, or formation level. The depth and width of the excavation for a tunnel on the broad gauge must, in both cases, be proportionately larger.

The annexed figure is a cross-section of the masonry of a tunnel, with the ballasting below the rails, which, of course, is such as is required where the tunnel is made through loose earth, only the arch above being required when made through hard rock, the side walls of the tunnel, in this case, being usually cut out of the embedded rock.



A TABLE

OF THE DIMENSIONS OF SEVERAL EXISTING TUNNELS.

Names of Railways and of Tunnels.		Length in Yards.	Width in Feet.	Height in Feet.	Sha 's.
London and Birmingham.	Primrose Hill	1120	22	22	5
	Weedon.......	418	—	—	
	Kilsby........	2398	—	—	10
Cromford and High Peak—Buxton.		580	21	16	
Great Western—The Box Tunnel...		3227	35	29	13
Manchester and Leeds.	Littleborough...	2869	24	21½	
North Midland	Claycross............	1806	22	21	
	Melford.............	836	—	—	
London and Brighton	Merstham..	1780			
	Balcombe..	1192			
Sheffield and Manchester.	Five Tunnels Total Length ...	6245			
Chester and Holyhead.	Three Tunnels Total Length ...	2130			
South Western	Seven Tunnels Total Length ...	1998			

SECTION V.

SUPERELEVATION OF EXTERIOR RAIL IN CURVES.

THE superelevation of the exterior rail in curves, the radii of which are within certain limits, is absolutely necessary to counteract the centrifugal force caused by the velocity of the train, since all moving bodies have a tendency to continue their motion in a direct line. From this cause the carriages of a railway train are driven towards the exterior rail, and would finally be thrown off the rails, were it not for the conical inclination of the tire and the flanges of the wheels.

Let W = weight of the moving body or train, V = its velocity per second, R = radius of the curve, and g = force of gravity at the earth's surface; then, per Dynamics, the centrifugal force

$$f = \frac{W V^2}{g R}. \qquad (1.)$$

When R = 1 mile = 5280 feet, V = velocity* = 60 miles per hour = 88 feet per second, and $g = 32\frac{1}{6}$ feet, then

$$f = \frac{W \times 88^2}{32\frac{1}{6} \times 5280} = \frac{1}{22} W,$$

that is, the centrifugal force that urges the moving body to leave the curve, in this case, is $\frac{1}{22}$ of its weight.

This force is in most cases counteracted by the conical inclination of the tire of the wheels, each pair of which is firmly fixed on the axle that turns with them, the inclination of the tire being usually about $\frac{1}{2}$ an inch in the whole breadth of the wheel, which is $3\frac{1}{2}$

* The great velocities, which some contemplate as likely to be attained on railways, would greatly augment the centrifugal force of the train, when moving in a curve; since that force varies as $\frac{V^2}{R}$, or as V^2 for the same curve: thus, for a velocity of 120 miles per hour, on a curve of 1 mile radius, we shall have

$$f = \frac{W \times 176^2}{32\frac{1}{6} \times 5280} = \frac{2}{11} W,$$

or the centrifugal force, in this case, is nearly $\frac{1}{5}$ of the weight of the whole train. It must, therefore, be evident that such a velocity, or even one of 100 miles per hour, must be extremely dangerous, in case of an accident throwing the train off the rails, especially on an embanked curve. Besides, the resistance of the air varies as V^2, which, in case of high winds opposed to the direction of the train, must be very considerable to a train having such velocity; and consequently its engine would require a power vastly superior to those at present employed.

inches, or about 1 in 7. This inclination of the tire together with the lateral play of the flanges of $\frac{1}{2}$ an inch on each side, and the centrifugal force impelling the carriages of the train, when moving in a curve, towards the exterior rail, enlarge the diameter of the exterior wheel, and diminish that of the interior; thus causing the train to roll on conical surfaces, which necessarily produces a centripetal force, the centre of which force is the vertex of the cone, of which the increased and diminished diameters of each pair of wheels are sections.

Let d be the outer diameter of the wheels, δ the increment and consequently the decrement that the diameters of the exterior and interior wheels respectively receive, through the joint action of the centrifugal force and the inclination of the tire: then under these circumstances the respective diameters of the exterior and interior wheels will be

$$d+\delta \text{ and } d-\delta\,;$$

also, if R′ be the radius of a circle which the centre of the carriage would describe in consequence of the inclination of the tire of the wheels, and b the breadth of the road or gauge: then $R' + \frac{1}{2}b$ and $R'-\frac{1}{2}b$, are the radii which would be respectively described by the exterior and interior wheels; and by similar triangles

$$d+\delta : d-\delta :: R'+\tfrac{1}{2}b : R'-\tfrac{1}{2}b,$$

whence $$d : \delta :: 2\,R' : b, \text{ and}$$

$$R' = \frac{b\,d}{2\,\delta}.$$

or, if $\frac{1}{n}$ = inclination of the tire, and Δ the deviation of the wheels, the increment or decrement $\delta = \frac{2\,\Delta}{n}$; and, by substitution,

$$R' = \frac{b\,d\,n}{4\,\Delta} \qquad (2.)$$

Also W and V representing the weight and velocity of the train, as in (1.), and g the force of gravity, the centripetal force corresponding to the radius R′ will be

$$f' = \frac{W\,V^2}{g\,R'}. \qquad (3.)$$

or, by substituting the value of R′ from (2.)

$$f' = \frac{W\,V^2}{g} \times \frac{4\,\Delta}{b\,d\,n}. \qquad (4.)$$

Since the forces f and f', (1.) and (3.), act in contrary directions, they will hold each other in equilibrium when they become equal, and the train will cease to have a tendency to leave the curve; this takes place when

$$\frac{W V^2}{g R} = \frac{W V^2}{g R'}$$

$$\text{or } R = R'.$$

Also from (1.) and (4.)

$$\frac{W V^2}{g R} = \frac{W V^2}{g} \times \frac{4 \Delta}{b d n};$$

whence

$$\Delta = \frac{b d n}{4 R}; \qquad (5.)$$

which is the deviation required to produce the equilibrium between the centripetal and centrifugal forces of the train. Therefore since $R = R'$, *i. e.* the vertex of the imaginary cone, of which each pair of wheels are sections, will coincide with the centre of the curve, there will in consequence be no dragging of either of the wheels on the rail.

If in (2.) $D = 3$ feet, $b = 4$ ft. $8\frac{1}{2}$ in. $= 4.7$ feet $=$ breadth of the narrow gauge, $\frac{1}{n} = \frac{1}{7}$, and $\Delta = \frac{1}{2}$ an inch, there will result for the least possible radius of curvature, in which the two forces balance each other, supposing the two rails to be exactly level,

$$R' = \frac{b d n}{4 \Delta} = 4.7 \times 3 \times 7 \div 4 \times \tfrac{1}{24} = 592 \textit{ feet}. \qquad (6.)$$

But as there might occur an accidental depression of the exterior rail, which would cause the flange of the wheel to rub the rail on that side, it is thought advisable, for the sake of greater safety, to limit the value of R' to not less than 1000 or 1500 feet, or what amounts to the same thing, to take the lateral deviation of the train at about $\frac{1}{2}$ or $\frac{1}{3}$ of that taken in (6.), or at about $\frac{1}{4}$ or $\frac{1}{6}$ of an inch, thus producing a less disagreeable displacement of the carriages of the train.

From what has been already shown it will at once appear that, in curves of less than 1000, or 1500, feet radius, a superelevation of the exterior rail will be necessary to counteract the excess of the centrifugal force above the centripetal force.

Let the superelevation of the exterior rail be denoted by x, and since b expresses the width of the way, the plane on which the train moves will be inclined $\frac{x}{b}$ to radius $= 1$, and therefore the gravity of the train will draw it to the interior rail with the force

$$f'' = \frac{W x}{b} \qquad (7.)$$

This force together with the centripetal force, due to the deviation of the carriages of the train on the rails, must hold the centrifugal force in equilibrium: we shall therefore have from equations (1.), (3.), and (7.)

$$\frac{W x}{b} + \frac{W V^2}{g R'} = \frac{W V^2}{g R};$$

whence

$$x = \frac{b V^2}{g}\left(\frac{1}{R} - \frac{1}{R'}\right),$$

which is the *Formula for the superelevation of the exterior rail.*

In which if $R = 900$ feet, $R' = 1000$ feet, $b = 4.7$ feet, $g = 32\frac{1}{6}$ feet, and $V =$ maximum velocity $= 60$ miles per hour $= 88$ feet per second, there will result $x = \frac{4.7 \times 88^2}{32\frac{1}{6}}\left(\frac{1}{900} - \frac{1}{1000}\right) = .1264$ of a foot $= 1.516$ inches, or about $1\frac{1}{2}$ inches.

This highly important Formula is due to the *Comte de Pambour;* see Chap. XVIII. Sect. III. page 534. of his valuable work on *Locomotive Engines;* to which I would refer those who wish for extensive information on these subjects : also to *Cresy's Encyclopædia of Engineering;* to *Hann on the Steam-Engine, with practical Rules for the use of Engineers;* and to *Tate's Exercises on Mechanics and Theory of the Steam-Engine,* &c.; which two last works cannot be too highly recommended on account of their important bearing on subjects of *real* practical utility.

No. 1. Reduction for each Chain for the following Angles of Elevation.

Angles.	Reduction in Links.
3° 0′	0.137
3 30	0.187
4 0	0.244
4 30	0.308
5 0	0.381
5 30	0.460
6 0	0.548
6 30	0.643
7 0	0.745
7 30	0.856
8 0	0.973
8 30	1.098
9 0	1.231
9 30	1.371
10 0	1.519
10 30	1.675
11 0	1.837
11 30	2.008
12 0	2.185
12 30	2.370
13 0	2.553
13 30	2.763
14 0	2.970
14 30	3.185
15 0	3.407
15 30	3.637
16 0	3.874
16 30	4.118
17 0	4.370
17 30	4.628
18 0	4.894
18 30	5.168
19 0	5.448
19 30	5.736
20 0	6.031
20 30	6.333
21 0	6.642
21 30	6.958
22 0	7.282
22 30	7.612
23 0	7.950
23 30	8.294
24 0	8.645
24 30	9.004
25 0	9.369
25 30	9.741
26 0	10 121
26 30	10.507
27 0	10.899
27 30	11.299
28 0	11.705
28 30	12.118
29 0	12.538
29 30	12.964
30 0	13.397

No. 2. Difference between Apparent and True Level for distances in Chains. Correction in Decimals of Feet.

Distances in Chains.	For Curvature.	For Refraction.	For Curvature and Refraction.
3	.0009	.0001	.0008
3½	.0013	.0002	.0011
4	0017	.0002	.0015
4½	.0021	.0003	.0018
5	.0026	.0004	.0022
5½	.0031	.0004	.0027
6	.0037	.0005	.0032
6½	.0044	.0006	.0038
7	.0051	.0007	.0044
7½	.0058	.0008	.0050
8	.0067	.0010	.0057
8½	.0075	.0011	.0064
9	.0084	.0012	.0072
9½	.0094	.0013	.0081
10	.0104	.0015	.0089
10½	.0115	.0016	.0099
11	.0126	.0018	.0108
11½	.0138	.0020	.0118
12	.0150	.0021	.0129
12½	.0163	.0023	.0140
13	.0176	.0025	.0151
13½	.0190	.0027	.0163
14	.0204	.0029	.0175
14½	.0219	.0031	.0188
15	.0234	.0033	.0201
15½	.0250	.0036	.0214
16	.0267	.0038	.0229
16½	.0284	.0041	.0243
17	.0301	.0043	.0258
17½	.0319	.0046	.0273
18	.0338	.0048	0290
18½	.0357	.0051	.0360
19	.0376	.0054	.0322
19½	.0396	.0056	.0340
20	.0417	.0060	.0357
20½	.0438	.0063	.0375
21	.0459	.0066	.0393
21½	.0481	.0069	.0412
22	.0504	.0072	.0432
22½	.0527	.0075	.0452
23	.0551	.0079	.0472
23½	.0575	.0082	.0493
24	.0600	.0086	.0514
24½	.0625	.0089	.0536
25	.0651	.0093	.0558
25½	.0677	.0097	.0580
26	.0704	.0100	.0604
26½	.0731	.0104	.0627
27	.0759	.0108	.0651
27½	.0788	.0113	.0675
28	.0817	.0117	.0700
28½	.0846	.0121	.0725
29	.0876	.0125	.0751
29½	.0906	.0129	.0777
30	.0937	.0134	.0803

No. 3. Difference between Apparent and True Level for distances in Miles. Correction in Feet and Decimals.

Distances in Miles.	For Curvature.	For Refraction.	For Curvature and Refraction.
¼	.042	.006	.036
½	.167	.024	.143
¾	.375	.054	.321
1	.667	.095	.572
1½	1.501	.215	1.286
2	2.668	·381	2.287
2½	4.169	.596	3.573
3	6.003	.856	5.147
3½	8.171	1.167	7.004
4	10.672	1.525	9.147
4½	13.547	1.930	11.517
5	16.675	2.382	14.293
5½	20.177	2.882	17.295
6	24.012	3.430	20.582
6½	28.181	4.026	24.155
7	32.683	4.669	28.014
7½	37.519	5.360	32.159
8	42.688	6.098	36.590
8½	48.191	6.884	41.307
9	54.027	7.718	46.309
9½	60.197	8.600	51.597
10	66.700	9.529	57.171
10½	71.520	10.217	61.303
11	80 687	11.527	69.160
11½	88.167	12.594	75.573
12	96.000	13.714	82.286
12½	104.167	14.881	89.286
13	112.667	16.095	96.572
13½	121.500	17.357	104.143
14	130.667	18.666	112.001
14½	140.167	20.024	120.143
15	150.000	21.428	128.572
15½	160.167	22.881	137.286
16	170.667	24.381	146.286
16½	181.500	25.928	155.572
17	192.667	27.524	165.143
17½	204.167	29.167	175.000
18	216.000	30.857	185.143
18½	228.167	32.581	195.586
19	240.667	34.381	206.286
19½	253.500	36.214	217.286
20	266.667	38.075	228.592
21	294.000	42.000	252.000
22	322.667	46.095	276.572
23	352.667	50.381	302.286
24	384.000	54.857	329.143
25	416.667	58.095	358.572
26	450.667	64.381	386.286
27	486.000	69.428	416.572
28	522.667	74.667	448.000
29	560.667	80.095	480.572
30	600.000	85.714	514.286

No. 4.
Offsets or Ordinates at the end of the first Chain from tangent-point of Railway Curves.

Radius of Curve in Chains.	Offsets in Inches and Decimals.	Radius of Curve in Chains.	Offsets in Inches and Decimals.	Radius of Curve in Chains.	Offsets in Inches and Decimals.	Radius of Curve in Chains.	Offsets in Inches and Decimals.
15	26.4000	65	6.0923	125	3.1680	225	1.7600
16	24.7500	66	6.0000	126	3.1428	230	1.7217
17	23.2941	67	5.9104	128	3.0937	235	1.6851
18	22.0000	68	5.8235	130	3.0461	240	1.6500
19	20.8421	69	5.7391	132	3.0000	245	1.6163
20	19.8000	70	5.6571	134	2.9552	250	1.5840
21	18.8571	71	5.5774	135	2.9333	255	1.5529
22	18.0000	72	5.5000	136	2.9117	260	1.5231
23	17.2174	73	5.4246	138	2.8645	265	1.4943
24	16.5000	74	5.3513	140	2.8285	270	1.4667
25	15.8400	75	5.2800	142	2.7887	275	1.4400
26	15.2307	76	5.2105	144	2.7500	280	1.4143
27	14.6666	77	5.1428	145	2.7310	285	1.3995
28	14.1428	78	5.0769	146	2.7123	290	1.3655
29	13.6551	79	5.0126	148	2.6756	295	1.3423
30	13.2000	80	4.9500	150	2.6400	300	1.3200
31	12.7742	81	4.8889	152	2.6052	305	1.2983
32	12.3750	82	4.8292	154	2.5714	310	1.2774
33	12.0000	83	4.7711	155	2.5548	315	1.2571
34	11.6470	84	4.7143	156	2.5384	320	1.2375
35	11.3142	85	4.6588	158	2.5063	325	1.2184
36	11.0000	86	4.6046	160	2.4750	330	1.2000
37	10.7026	87	4.5517	162	2.4444	335	1.1821
38	10.4210	88	4.5000	164	2.4146	340	1.1647
39	10.1538	89	4.4496	165	2.4000	345	1.1478
40	9.9000	90	4.4000	166	2.3855	350	1.1314
41	9.6588	91	4.3516	168	2.3571	355	1.1155
42	9.4285	92	4.3043	170	2.3294	360	1.1000
43	9.2093	93	4.2581	172	2.3023	365	1.0809
44	9.0000	94	4.2128	174	2.2758	370	1.0703
45	8.8000	95	4.1684	175	2.2628	375	1.0560
46	8.6087	96	4.1250	176	2.2500	380	1.0421
47	8.4255	97	4.0825	178	2.2248	385	1.0285
48	8.2500	98	4.0408	180	2.2000	390	1.0154
49	8.0816	99	4.0000	182	2.1758	395	1.0025
50	7.9200	100	3.9600	184	2.1521	400	.9900
51	7.7647	102	3.8824	185	2.1405	410	.9659
52	7.6154	104	3.8077	186	2.1290	420	.9428
53	7.4717	105	3.7714	188	2.1064	430	.9209
54	7.3333	106	3.7358	190	2.0842	440	.9000
55	7.2000	108	3.6667	192	2.0625	450	.8800
56	7.0714	110	3.6000	194	2.0412	460	.8609
57	6.9473	112	3.5352	195	2.0307	470	.8425
58	6.8276	114	3.4736	196	2.0204	480	.8250
59	6.7118	115	3.4435	198	2.0000	490	.8081
60	6.6000	116	3.4138	200	1.9800	500	.7920
61	6.4918	118	3.3559	205	1.9317	510	.7765
62	6.3871	120	3.3000	210	1.8857	520	.7615
63	6.2857	122	3.2459	215	1.8418	530	.7472
64	6.1875	124	3.1935	220	1.8000	540	.7333

No. 5.
Horizontal Distances to an Unit's Height to the following Angles of Elevation.

Angle of Elevation.		Hor. Distance to one Perpendicular.
0°	5′	688
0	10	344
0	15	229
0	30	115
0	45	76
1	0	57
1	15	46
1	30	39
1	45	33
2	0	28
2	15	25
2	30	23
2	45	21
3	0	19
3	15	18
3	28	17
3	35	16
3	49	15
4	6	14
4	24	13
4	45	12
5	12	11
5	42	10
6	21	9
7	7	8
8	8	7
9	27	6
11	19	5
14	2	4
18	26	3
26	34	2
45	0	1

TH

RY EARTH-WORK TABLES.

No. 2.

A Table of Cubic Yards to be added for the Decimal Parts of a Foot, to be used with the general Table for the Calculation of Earthwork, when required.

$\frac{1}{10}(2a+b)$ or $\frac{1}{10}(2b+a)$	·1	·2	·3	·4	·5	·6	·7	·8	·9

62	6.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
63	6.2857	122	3.2459	215	1.8418	[illegible]	.747[illegible]		
64	6.1875	124	3.1935	220	1.8000	540	.7333		